Nonlinear Dynamics of Asymmetrically Coupled Microdisk Lasers

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***Abstract*—The operating regimes of coupled microdisk lasers are investigated using rate equation modeling. We found that unidirectional operation is possible for smaller currents than for single disks. However, we also found regimes of chaos.**

***Index Terms*—AOFF, microdisk lasers, opical memory, optical chaos**

I. Introduction

MICRODISK and/or microring lasers offer a promising solution for advanced optical signal processing because they have been demonstrated to operate as all-optical flip-flops in both coupled as single configurations [1-2]. Their behaviour is based on the bistable unidirectional operation which can be obtained at sufficiently high bias currents with a low coupling between clockwise (CW) and counterclockwise (CCW) fields. At the same time, thin film microdisk lasers with diameters as small as 7.5 μm have been shown to operate in continuous wave mode at currents as small as a few mA [3]. The combination of a small footprint, a low power consumption and the ability to integrate heterogeneously with passive optical circuits using silicon-on-insulator (SOI) technology, makes these thin film microdisk lasers promising candidates as key components in the next generation of integrated photonic circuits.

A thorough mathematical and numerical analysis has already been performed for single microring (disk) lasers [4-5], but not yet for their coupled counterpart and little is known about the behavior of these coupled lasers except for the experimental results given in [1]. In this paper we present a detailed investigation on the dynamics of coupled ring lasers. We will demonstrate that at low currents the coupled lasers can act as chaos sources and by increasing the current we find oscillating, bidirectional and unidirectional operation.

II. Model

A. Single Microdisk Laser

Each microdisk laser accommodates two counter-propagating whispering gallery modes with slowly varying complex amplitudes E^+ (CCW) and E^- (CW). The dynamics of each mode can be described by a simple complex differential equation:

$$\frac{dE^{\pm}}{dt} = \frac{1}{2}(1-j\alpha)\left(G^{\pm} - \frac{1}{\tau_p}\right)E^{\pm} + CE^{\mp} = \Phi^{\pm}(E^{\pm}, E^{\mp}) \quad (1)$$

with α the linewidth enhancement factor, τ_p the photon lifetime and C the inter-modal coupling due to surface roughness or external reflections. The gain coefficient $G^{\pm}$ includes nonlinear gain suppression ϵ_{NL}:

$$G^{\pm} = \frac{\Gamma g_n(N-N_0)}{1+\Gamma\epsilon_{NL}(|E^{\pm}|^2 + 2|E^{\mp}|^2)} \quad (2)$$

with g_n the differential gain, N_0 transparency threshold and Γ the confinement factor. Carrier dynamics can be described by a simple model including only a linear spontaneous emission term:

$$\frac{dN}{dt} = \eta\frac{I}{q} - \frac{N}{\tau_c} - G^+|E^+|^2 - G^-|E^-|^2 \quad (3)$$

with I the bias current, η the current efficiency and τ_c the carrier lifetime. Solving this system will yield a bidirectional, oscillating and unidirectional regime respectively for increasing current.

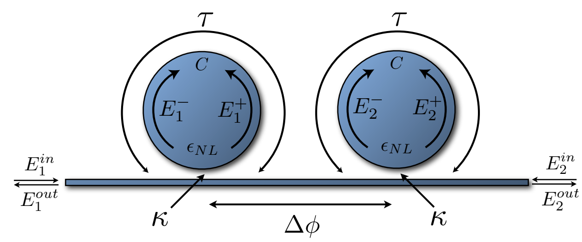

Fig. 1. Schematic of the structure of two coupled disks

B. Asymmetrically Coupled Microdisk laser

By connecting two identical microdisk lasers by a single waveguide (Figure 1), the system can be described by an extended set of equations:

$$\frac{dE_1^+}{dt} = \Phi^+(E_1^+, E_1^-) \quad (4)$$

$$\frac{dE_1^-}{dt} = \Phi^-(E_1^-, E_1^+) - \kappa^2\tau\exp(j\Delta\phi)E_2^- \quad (5)$$

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and similar equations for the second disk. Subscripts identify the disk and superscripts denote the optical mode in the disk. We introduce a few new parameters: κ is the field-coupling strength between disk and waveguide per unit time, τ quantifies the roundtrip time for a single disk and $\Delta\phi$ denotes the phase-change caused by the optical length of the connecting waveguide.

III. Operating Regimes

We can clearly observe the different operating regimes when depicting the output at the two waveguide ends (Figure 2). The low current regime shows chaotic behavior while for higher currents oscillating and unidirectional operation occurs.

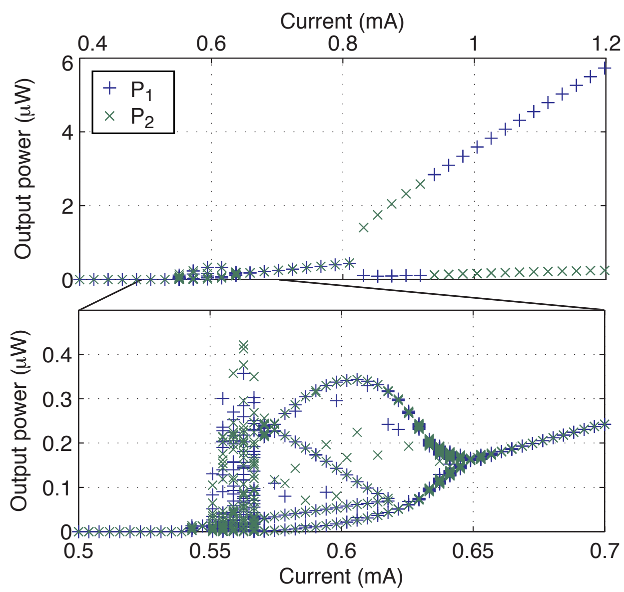

Fig. 2. Bifurcation diagram for coupled microdisk lasers with coupling κ of $4\%/\tau$ and $\Delta\phi$= 0.

A. Low Current Regime: Optical Chaos

Due to the interplay between the inter-modal coupling C and the coupling between disks $\kappa^2\tau$, optical chaos occurs even though we did not take into account any time-delay between both disks (which is a common source for chaotic behavior).

Using a Lyapunov approach [6], we quantified the level of chaos as a function of different parameters in the system. The most remarkable result in this analysis is found by varying the phase-change $\Delta\phi$ between both disks. Figure 3 shows how the largest Lyapunov exponent evolves as a function of $\Delta\phi$. The curve is periodic with period π and shows two local maxima, one local minimum and a stable region with no chaos. This particular shape opens up many exciting possibilities. One could for example design an optical chaos source with a maximum Lyapunov number by fixing the distance between both microdisks such that $\Delta\phi$ is at a maximum. Due to the parabolic shape near these maxima, the device will be relatively insensitive to variations in wavelength or the waveguide's optical length.

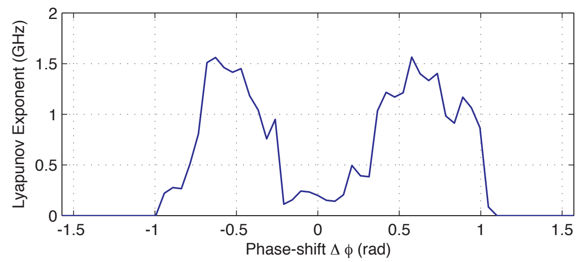

Fig. 3. Largest Lyapunov exponent as a function of phase shift $\Delta\phi$ for a current of 0.6 mA/disk.

B. High Current Regime: All-Optical Flip-Flop Operation

For high currents, the coupled system has three stable states: one bidirectional state and two unidirectional states. For each state both disks are in the unidirectional regime seperately but the actual dominant mode in each disk will determine the overall state. If both disks have the same dominant mode, eg. the + state, the overall state will be unidirectional. If both dominant modes are counter-propagating (E_1^- and E_2^+ dominant), the device will be in a bidirectional regime.

The two unidirectional states allow this device to be operated as a flip-flop and we can compare its performance to that of a single microdisk flip-flop. Due to destructive interference between suppressed modes, the coupled design exhibits extinction ratios of 20 dB where single disks only reach 12 dB for the same current and unidirectional behavior is achieved at lower bias currents (0.8 mA/disk versus 1.5 mA). Switching between states in the coupled configuration occurs approximately 15% slower (50 ps versus 60 ps) and the minimal required switching energy is doubled.

IV. Conclusion

Using coupled rate equations, we have analysed numerically the behavior of coupled disk lasers and compared it with that of single disk lasers. Unidirectional behavior in coupled ring lasers is found for smaller currents (or larger coupling between clockwise and counterclockwise fields) and with larger extinction ratios (20 dB) than in single ring lasers. Moreover, a chaotic output can be achieved if the phase of the coupling between the rings/disks is within certain ranges.

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