Coupling Coefficients in Gain-Coupled DFB Lasers: Inherent Compromise Between Coupling Strength and Loss

K. David, J. Buus, G. Morthier, and R. Baets

Abstract—A theoretical analysis of the gain-coupling coefficient $\kappa_{gain} L$ for DFB lasers with first-order, rectangular, gain, or loss gratings is presented. For the structure with gain grating, the dependence of $\kappa_{gain}$ on the modal gain $g_{mod}$ has been taken into account for the first time. In both structures, an inherent compromise between coupling strength and extra modal loss is found. The results show that significant $\kappa_{gain} L$ values are feasible, allowing to benefit from the unique features of gain-coupled DFB lasers.

I. Introduction

RecentlY gain-coupled DFB lasers have attracted considerable interest [1]–[3] because of some remarkable advantages. As was already shown by Kogelnik and Shank [4], the degeneracy problem of AR-coated DFB lasers is eased by the introduction of pure gain coupling, and one finds one lasing mode exactly at the Bragg frequency. If combined index and gain coupling is considered, a significant improvement in terms of reduced spatial hole burning and increased threshold gain difference as compared to index-coupled DFB lasers is still found [5], [6]. Even for imperfect AR coatings, relevant improvements are found [2], [7], [8]. This shows that AR coatings are not indispensable for gain-coupled DFB lasers. In addition, the first experimental [3] and theoretical [2] results show a potential for lower feedback sensitivity compared to other DFB lasers.

One important point for the fabrication of these attractive gain-coupled DFB lasers is the calculation and realization of the gain-coupling coefficient $\kappa_{gain}$. Basically, there are two possibilities, both of which have been realized experimentally [1], [9], [7]: a periodic variation of the gain or of the loss along the longitudinal axis of the laser. The coupling coefficient $\kappa_{gain}$ of the gain grating has been examined in [9], but there the dependence of $\kappa_{gain}$ on the modal gain $g_{mod}$ was not taken into account, and therefore these results should be treated with care.

In this paper, we will first briefly review the calculation of the index-coupling coefficient $\kappa_{index}$. Based on this, the gain-coupling coefficient $\kappa_{gain}$ will be derived. A mechanism, limiting the $\kappa_{gain} L$ value, will be studied. Next, the relation between $\kappa_{gain}$ and losses in a loss grating will be presented.

II. Results

A. Coupling Coefficients of Index Gratings: A Review

For clarity and simplicity, we consider first-order rectangular gratings, but the results are valid for higher order gratings if radiation losses are neglected. The derivation of the coupled-mode equations for a slab waveguide with grating perturbation gives the well-known result for $\kappa_{index}$ [10]:

$$\kappa_{index} = \frac{k_0^2}{2 \beta} \int_{\text{index grating}} A(x) \Delta \varepsilon E_0^2(x) \, dx$$

(1)

where $k_0 = 2 \pi / \lambda_0$ is the free-space wavenumber, $A(x)$ is the appropriate Fourier component of the perturbation along the longitudinal coordinate, $\Delta \varepsilon = \Delta \varepsilon_{index} = n_2^* - n_2^0$, $n_1$ and $n_2$ being the refractive indexes above and below the grating interface, and we have the assumption of normalised optical fields $(E(x))$. If $A(x)$ is constant, (1) can be rewritten as

$$\kappa_{index} = \frac{k_0^2}{2 \beta} A \Delta \varepsilon \Gamma_{E\kappa}$$

(2)

where $\Gamma_{E\kappa}$ is the confinement factor of the rectangular index grating. $A$ is given by [10]

$$A = \frac{\sin(\pi \Lambda_\kappa / \Lambda)}{\pi}$$

(3)

with $\Lambda_\kappa$ and $\Lambda$ as explained in Fig. 1.

B. Coupling Coefficients of Gain Gratings

The analysis given above can be repeated for pure gain coupling. $\Delta \varepsilon$ is now given by $\Delta \varepsilon_{gain} = n_\rho \Delta g / k_0$, and $\Gamma_{E\kappa}$ becomes $\Gamma_{E\kappa}$, the confinement factor of the gain grating. $n_\rho$
is the effective refractive index \( \beta = \pi \alpha_k^{-1} \). This results in

\[
\kappa_{\text{gain}} = \frac{\sin \left( \frac{\pi \lambda_0}{\lambda} \right)}{2\pi} \Delta g \Gamma_{\text{ee}}.
\]

(4)

\( \Delta g \) is given by

\[
\Delta g = g_{\text{mat}} - \alpha_{\text{int}} + \alpha_{\text{cl}}^*.
\]

(5)

with \( g_{\text{mat}} \) being the material gain, \( \alpha_{\text{int}} \) the average internal losses in the active layer, and \( \alpha_{\text{cl}} \) the loss in the cladding layer in the grating region; see Fig. 1. \( \alpha_{\text{int}} \) and \( \alpha_{\text{cl}}^* \) are assumed to be equal for simplicity. Otherwise, we would have additional loss coupling, which will be treated in the next section. To simplify the following equations, we add all losses into an effective modal loss \( \alpha_{\text{mod}} \) given by

\[
\alpha_{\text{mod}} = \Gamma_{\text{ac}} + \frac{1 - \Delta_1}{\Delta_1^*} \Gamma_{\text{ee}} \alpha_{\text{int}} + \Gamma_{\text{ee}}^* \alpha_{\text{cl}}
\]

(6)

with \( \Gamma_{\text{ee}}^* \) the confinement factor of the cladding layers, \( \Gamma_{\text{ac}} \) that of the active layer, and \( \alpha_{\text{cl}}^* \) the losses in the cladding layers. \( \Gamma_{\text{ac}} \) is chosen such that the average gain caused by the perturbation averages to zero. The material gain \( g_{\text{mat}} \) is connected to the modal gain \( g_{\text{mod}} \) by

\[
g_{\text{mod}} = \Gamma_{\text{ac}} g_{\text{mat}} - \alpha_{\text{mod}}.
\]

(7)

As a first step, we neglect all losses, and it follows from (4), (5), and (7) that

\[
\kappa_{\text{gain}} = \frac{c}{2} \frac{\sin \left( \frac{\pi \lambda_0}{\lambda} \right)}{\Gamma_{\text{ac}}^*} \Gamma_{\text{ee}}
\]

(8)

with

\[
c = \frac{\pi}{\Gamma_{\text{ee}}^*} \Gamma_{\text{ac}}
\]

(9)

c is a geometrical constant, introduced to simplify the following expressions. Equation (8) shows how the gain-coupling coefficient is related to the modal gain, and hence the material gain.

The threshold (and above threshold) operation of the lasers is a function of the coupling strength. Hence, \( g_{\text{mod}} \) at threshold (written as \( g_{\text{modh}} \) and denoted as \( 2 \sigma_0 \) in [4]) is a function of \( \kappa_{\text{gain}} \). This relation is shown as curve \( b \) in Fig. 2. The intersection point \( A \) is the operation point at and above threshold. With the help of [4, eq. (19)] and the fact that for pure gain coupling the Bragg deviation is zero, it is straightforward to derive an analytical expression of the operation point. The sign ambiguity in [4, eq. (19)] is resolved by the fact that the minus sign corresponds to the solution where the gain maxima of the gain grating and the maxima of the standing-wave pattern in the laser show maximum overlap. The plus sign corresponds to minimum overlap, i.e., loss coupling [11]. The operation point is now given by

\[
g_{\text{modh}} = c \left( g_{\text{modh}} + \alpha_{\text{mod}} \right) \cdot \cosh \left( \frac{g_{\text{modh}}}{2} \right) = \frac{c}{2} \frac{\kappa_{\text{gain}} L \left( c = 1 \right) = 1}{1 - c^2}.
\]

(10)

It is interesting to note that \( g_{\text{modh}} \), and hence \( \kappa_{\text{gain}} L \) as well, are independent of \( L \). Equation (9) gives, with \( \Delta_1 / \Delta_1^* \Gamma_{\text{ee}} \leq \Gamma_{\text{ac}} \), the maximum \( c \): \( c = 1 \), and hence with (10), the maximum coupling: \( \kappa_{\text{gain}} L (c = 1) = 1 \).

\[\text{Fig. 2. Schematic relation between } \kappa_{\text{gain}} L \text{ and } g_{\text{modh}} L.\]

\[\text{Fig. 3. } \kappa_{\text{gain}} L (L = 300 \mu m) \text{ as a function of extra intensity losses limiting the } \kappa_{\text{gain}} L \text{ value. For the curves with full lines (gain grating), the extra modal loss equals } \alpha_{\text{modh}} \text{, and for the curves with dashed lines (loss grating), the extra modal loss is set equal to } \left( \Delta_1 / \Delta_1^* \right) \left( 2 \pi / \sin \left( \pi \Delta_1 / \lambda \right) \right) \kappa_{\text{gain}}. \text{ For a direct comparison, } \Gamma_{\text{ac}}^* \alpha_{\text{ac}} \text{ has to be added to the dashed lines.}\]

\[\kappa_{\text{gain}} L \text{ can be increased by a parallel shift of curve } a \text{ to, e.g., curve } a^* \text{ in Fig. 2. This shift corresponds to the introduction of the formerly neglected losses. Equation (8) now becomes}\]

\[
\kappa_{\text{gain}} = \frac{c}{2} \left( g_{\text{modh}} + \alpha_{\text{mod}} \right).
\]

(11)

This now leads to the following equation for the operation point:

\[
g_{\text{modh}} = c \left( g_{\text{modh}} + \alpha_{\text{mod}} \right) \cdot \cosh \left( \frac{g_{\text{modh}}}{2} \right) - \left( \frac{c}{2} \frac{\kappa_{\text{gain}} L \left( c = 1 \right) = 1}{1 - c^2} \right)^{1/2} L.
\]

(12)

The numerical solution of (12) can be seen in Fig. 3 for four values of \( c \). This shows an inherent compromise between \( \kappa_{\text{gain}} \) and extra modal losses.

The present analysis was limited to the case of pure gain coupling with one specific grating shape and with radiation effects being neglected. This has allowed us to derive simple results. Numerically, it is, of course, possible to include index coupling, other grating shapes, radiation losses, and facet reflectivities.

C. Coupling Coefficients of Loss Gratings

The second possibility of realizing gain coupling, a periodic variation of losses along the laser cavity, is sketched in
Fig. 4. Schematic structure of loss-coupled DFB laser.

For clarity and without loss of generality, we set all losses besides the ones in the active layer $\alpha_{act}$ and the periodic losses $\alpha_{per}$ to zero. $\alpha_{per}$ loss values of the order of 10 000/cm can be realized. An important difference with the gain grating is that the coupling coefficient is now independent of $g_{mod}$. For a first-order rectangular grating, $\kappa_{gain}$ is given by (4) with $\Delta g = \alpha_{per}$. Large $\kappa_{gain}$ values should be possible just by choosing an appropriate $\Gamma_{ext}$; but again, an inherent compromise with increased modal losses has to be considered:

$$g_{mod} = \Gamma_{av} (g_{mod} - \alpha_{int}) - \frac{\lambda_l}{\Lambda} \frac{2\pi}{\sin (\pi \kappa_{gain} / \Lambda)}.$$  

(13)

The reduction of the extra modal losses by decreasing $\lambda_l / \Lambda$ can be seen in Fig. 3.

In Fig. 3, it can also be seen that for smaller $\kappa_{gain} L$ values, the gain-coupled DFB, and for large $\kappa_{gain} L$ values, the loss-coupled DFB, is advantageous in terms of lower additional losses. For a direct comparison, it should be remembered that in Fig. 3, a loss of $\Gamma_{av} \alpha_{int}$ has to be added for the loss-coupled DFB.

III. Conclusions

The coupling coefficient $\kappa_{gain}$ of gain-coupled DFB lasers, a most attractive DFB with unique performance features, has been examined. For a gain grating, the dependence of $\kappa_{gain}$ on the modal gain $g_{mod}$ has been taken into account self-consistently. This is shown to be an important dependence which is not present in pure index or loss gratings. The $\kappa_{gain}$ has also been given for a loss grating for which the losses can be reduced by decreasing its duty cycle. In both cases, an inherent compromise between loss and coupling strength is found, i.e., possible $\kappa_{gain} L$ values are related to the loss. For both gratings, the presented basic ideas are of significant interest for the design of optimized gain-coupled DFB lasers.

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REFERENCES


