# Tristable All-Optical Flip-Flop using Coupled Nonlinear Cavities

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Abstract—We demonstrate symmetry breaking in a multistable system composed of three coupled nonlinear cavities. Using positive pulses it is possible to switch between the asymmetric states, thus resulting in tristable flip-flop behaviour. Coupledmode theory gives analytical results and precise insight into the intricate dynamical behaviour.

Index Terms—All-optical flip-flops, Coupled-mode theory, Nonlinear cavities

# I. INTRODUCTION

All-optical flip-flops will play an important role in future packet or burst switched networks. They act as optical memory elements that temporarily store the routing information of the header [1]. Most concepts for all-optical flip-flops are based on the switching between two different states [2]–[4]. Advanced all-optical signal processing however will require switching between more than two outputs. Only very few concepts with more than two stable states have been proposed [5]. In addition most existing concepts are difficult to extend to multistable switching.

Here we show that nonlinear effects can break the symmetry in a non-dissipative passive Kerr system with three resonators, leading to multiple stable states. Switching between the different asymmetric states is possible by using positive optical pulses, so one obtains a *multistable* all-optical flip-flop. The use of symmetry breaking for the more conventional *bistable* flip-flop operation is investigated in [3].

In Figure 1 the structure under study is depicted. Three equal nonlinear cavities are coupled with each other. Continuous wave (CW) light is injected with the same phase in the three input ports. The proposed scheme can be implemented in a photonic crystal device. However, other systems e.g. ring resonators are possible as well.

### II. STATIC BEHAVIOUR

We apply coupled-mode theory on the structure of Figure 1 and write the following equation for the time dependence of the amplitude  $a_1$  of the resonance mode of the first cavity:

$$\frac{\mathrm{d}a_1}{\mathrm{d}t} = \left[j\left(\omega_0 + \delta\omega_1\right) - \frac{1}{\tau}\right]a_1 + df_1 + db_4 + df_6$$



Fig. 1. The structure with three nonlinear cavities and six waveguides.

with similar equations for the other cavities. Here  $d = j\sqrt{2/3\tau} \exp(j\phi/2)$  and  $\phi$  represents the phase that depends on the waveguide length and reflection properties. We assume that the three cavities have the same resonant mode with sufficient symmetry and center frequency  $\omega_0$ . Their nonlinear frequency shift is given by  $\delta\omega_i = -|a_i|^2/P_0\tau^2$ . The forward and backward waveguide amplitudes are coupled by [6]:

$$f_4 = \exp(j\phi)b_4 + da_1$$
$$b_4 = \exp(j\phi)f_4 + da_2$$

and similar equations hold for the other forward and backward propagating waves. Assuming equal input powers from all sides (i.e.  $f_1 = f_2 = f_3$ ) and CW operation with frequency  $\omega$ , we can analytically find the requirements for which symmetry breaking is possible. For positive nonlinearities ( $\delta \omega_i < 0$ ), we find the following condition for the detuning:

$$\omega - \omega_0)\tau - \frac{2\cos(\phi) - 1}{3\sin(\phi)} > \frac{1}{\sqrt{3}}$$

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By solving the coupled-mode equations in steady-state conditions and applying a linear stability analysis to eliminate the unstable states, we can find the static solutions as a function of the input power. In Figure 2 the stable output states are depicted as a function of the input power. The unit is the 'characteristic nonlinear power'  $P_0$  of the cavities as defined in [7].

One can clearly observe that besides the symmetric solution, asymmetric solutions show up for a certain range of input powers. In region I, we distinguish solutions where two of the

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Fig. 2. The stable states of the output power as a function of the input power for a structure with  $\tau(\omega - \omega_0) = 0.1$  and  $\phi = -0.5$ .



Fig. 3. The stable states of the cavity energy as a function of the input power for a structure with  $\tau(\omega - \omega_0) = 0.1$  and  $\phi = -0.5$ .

outputs are equal and have a higher value than the third output which is low. By increasing the input power the two equal outputs will split up, resulting in region II where the three outputs are different and the symmetry breaking is complete. This state gradually transforms to region III with one of the three outputs high and the other two low. We will show in the next section that by using positive pulses, one can switch between these latter states resulting in all-optical flip-flop operation.

In Figure 3 the energy of the resonant modes in the cavities is depicted as a function of the input power. We can clearly observe a bistable structure for the symmetric solution. By remaining below the threshold at  $P_{in} \approx 4.5P_0$ , one can easily avoid the upper stable branch when switching between the asymmetric states.

# III. DYNAMIC BEHAVIOUR

In the third regime of Figure 2, one can switch between the different asymmetric solutions by applying a short pulse to two of the three inputs. This results in a state where the



Fig. 4. Switching between the three different output ports.

port where no pulse is injected has the high output power. The switching characteristics are depicted in Figure 4, where we switch between the three possible asymmetric output states. The time is expressed in units of the characteristic lifetime  $\tau$  of the cavity. We use a constant input power of  $3.3P_0$  and increase this for two of the cavities to  $3.5P_0$  during a time  $30\tau$ . The transient behaviour takes less than  $100\tau$ .

# IV. IMPLEMENTATION

One could implement the proposed structure using monopole photonic crystal cavities. If these cavities have a Qfactor of about  $10^3$  and the nonlinear material has a realistic nonlinear index  $n_2 = 1.5 \ 10^{-13} \ \text{cm}^2/\text{W}$ , we can expect a characteristic power  $P_0$  of about 20 mW [7]. In that case we should inject CW light of 66 mW and use pulses of 200 fJ with a duration of about 50 ps. The expected switching times would be around 130 ps. The characteristic power  $P_0$  is proportional to  $1/Q^2$ , so higher Q-factors result in lower input powers, but also higher values of the cavity lifetime  $\tau$ .

## V. CONCLUSION

We theoretically analyzed the conditions for symmetry breaking in three coupled nonlinear cavities. Switching between the asymmetric states demonstrates its use for all-optical tri-state flip-flop operation.

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