PART 11

OPTICAL PROPERTIES OF FILMS AND COATINGS
CHAPTER 42

OPTICAL PROPERTIES
OF FILMS AND COATINGS

J. A. Dobrowolski
Institute for Microstructural Sciences
National Research Council of Canada
Ottawa, Ontario, Canada

42.1 GLOSSARY

Quantities, Abbreviations

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>A</td>
<td>absorptance of layer system</td>
</tr>
<tr>
<td>BW</td>
<td>base width of a bandpass filter</td>
</tr>
<tr>
<td>D</td>
<td>spectral detectivity of a detector</td>
</tr>
<tr>
<td>DW</td>
<td>Debye-Waller factor (used in XUV coatings)</td>
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<tr>
<td>d</td>
<td>metric thickness of a layer</td>
</tr>
<tr>
<td>E</td>
<td>electric vector of electromagnetic field</td>
</tr>
<tr>
<td>FP</td>
<td>Fabry-Perot (interference filter)</td>
</tr>
<tr>
<td>H</td>
<td>magnetic vector of electromagnetic field</td>
</tr>
<tr>
<td>H</td>
<td>quarterwave layer of high refractive index</td>
</tr>
<tr>
<td>HW</td>
<td>halfwidth of a bandpass filter</td>
</tr>
<tr>
<td>I</td>
<td>intensity of light</td>
</tr>
<tr>
<td>k</td>
<td>extinction coefficient of layer or medium</td>
</tr>
<tr>
<td>L</td>
<td>quarterwave layer of low refractive index</td>
</tr>
<tr>
<td>L</td>
<td>number of layers in system</td>
</tr>
<tr>
<td>LIDT</td>
<td>laser-induced damage threshold</td>
</tr>
<tr>
<td>M</td>
<td>layer matrix</td>
</tr>
<tr>
<td>n</td>
<td>refractive index of layer or medium</td>
</tr>
<tr>
<td>nd</td>
<td>optical thickness of a layer</td>
</tr>
<tr>
<td>ñ</td>
<td>complex refractive index</td>
</tr>
<tr>
<td>P</td>
<td>degree of polarization</td>
</tr>
<tr>
<td>R</td>
<td>reflectance of layer system</td>
</tr>
<tr>
<td>r</td>
<td>amplitude reflection coefficient</td>
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\( S \) spectral intensity distribution
\( T \) transmittance of layer system
\( t \) amplitude transmission coefficient
\( \delta \) phase thickness of a layer
\( \varepsilon \) phase change on reflection or transmission
\( \eta \) effective refractive index
\( \theta \) angle of incidence
\( \theta_o \) angle of incidence in the incident medium
\( \lambda \) wavelength
\( \lambda_o \) reference or central wavelength
\( \mu^* \) effective index of a bandpass filter
\( \sigma \) combined effect of interface layer and roughness (in XUV coatings)
\( \sigma \) wavenumber
\( \tau \) internal transmittance of a substrate

Subscripts

\( ij \) where \( i,j = 1 \) or \( 2 \), for elements of layer matrix
\( j \) for \( j \)-th layer
\( m \) for medium
\( p \) for light polarized parallel to the plane of incidence
\( r \) for reflected light
\( s \) for light polarized perpendicular to the plane of incidence
\( s \) for substrate
\( t \) for transmitted light

42.2 INTRODUCTION

Scope of Chapter

In the broadest sense of the term, an optical filter is any device or material which is deliberately used to change the spectral intensity distribution or the state of polarization of the electromagnetic radiation incident upon it. The change in the spectral intensity distribution may or may not depend on the wavelength. The filter may act in transmission, in reflection, or both.

Filters can be based on many different physical phenomena, including absorption, refraction, interference, diffraction, scattering, and polarization. For a comprehensive review of this broader topic the interested reader is referred to the article entitled “Coatings and Filters” which appeared in the first edition of the *Handbook of Optics.*
This chapter deals only with filters that are based on absorption and interference of electromagnetic radiation in thin films. Optical thin-film coatings have numerous applications in many branches of science and technology and there are also many consumer products that use them. The spectral region covered in this chapter extends from about 0.003 to 300 μm (3 to 3×10⁵ nm), although the main emphasis is on filters for the visible and adjacent spectral regions.

The discussion in this chapter is largely confined to generic thin-film filters, such as antireflection coatings, cut-off filters, narrowband transmission or rejection filters, reflectors, beam splitters, and so forth. Filters for very specific applications, such as filters for colorimeters and other scientific instruments, color correction filters, and architectural coatings, are, as a rule, not treated. Of the filters described, many are available commercially while others are only research laboratory prototypes. This review does not cover thin-film filters whose properties can be changed by external electric or magnetic fields, temperature, or illumination level.

In this introductory section some general considerations on the use of optical filters are presented. In the following sections, the theory of optical multilayers and the methods for their deposition and characterization are briefly discussed. These sections are useful for gaining a proper understanding of the operation, advantages, and limitations of optical coatings. The remaining sections then describe the properties of various generic thin-film filters.

For further information on this subject, the interested reader is particularly encouraged to consult the books by Macleod and Rancourt.

**General Theory of Filters**

There are many different ways of describing the performance of optical coatings and filters. For example, transmission and reflection filters intended for visual applications are adequately described by a color name alone, or by reference to one of the several existing color systems (see Chap. 26). There also exist other specialized filter specifications for specific applications. However, the most complete information on the performance of a filter is provided by spectral transmittance, reflectance, absorptance, and optical density curves. This is the method adopted in this chapter.

Referring to Fig. 1, at a wavelength λ the normal incidence spectral transmittance \( T(\lambda) \)
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of a filter placed between two semi-infinite media is equal to the ratio of the light intensity of that wavelength transmitted \( I_T(\lambda) \) by the filter to that incident \( I_o(\lambda) \) upon it,

\[
T(\lambda) = \frac{I_T(\lambda)}{I_o(\lambda)}
\]

(1)

At nonnormal incidence, the component of the intensity perpendicular to the interface must be used in the preceding equation. The spectral reflectance \( R(\lambda) \) of a filter is defined in a similar way,

\[
R(\lambda) = \frac{I_R(\lambda)}{I_o(\lambda)}
\]

(2)

The relationship between the transmittance \( T(\lambda) \) and the density of a filter, sometimes also called the absorbance, is given by

\[
D(\lambda) = \log \frac{1}{T(\lambda)}
\]

(3)

In this chapter, transmittances and reflectances will be plotted using either linear or logarithmic scales. The logarithmic scale is particularly well suited whenever accurate information about the low-transmission or reflection region is to be conveyed. However, this is done at the expense of detail at the high end of the scale. Wavelengths are normally specified in micrometers (\( \mu m \)) because this is the most convenient unit for the spectral range covered in this chapter. In the following discussions, the dependence of the transmittance, reflectance, and absorptance on wavelength will be implicitly assumed.

Transmission and Reflection of Coatings on a Substrate

Many multilayer coatings are deposited onto a transparent or partially transparent substrate. Both the multilayer and the substrate contribute to the overall performance of the filter. For example, absorption in the substrate is frequently used to limit the transmission range of the filter. Reflectances at the filter interfaces need also to be considered. However, they can be reduced by antireflection coatings, or by cementing several components together.

In general, a filter can consist of multilayer coatings deposited onto one or both sides of a substrate. The overall transmittance \( T_{total} \) of a filter can be expressed in terms of the internal, or intrinsic transmittance \( \tau \) of the substrate and the transmittances \( T_1, T_2 \) and internal reflectances \( R_1, R_2 \) of each surface of the substrate (Fig. 1).

The internal transmittance \( \tau \) of a substrate is defined to be the ratio of the light intensity \( I'' \) just before reaching the second interface to the intensity \( I' \) just after entering the substrate (Fig. 1):

\[
\tau = \frac{I''}{I'}
\]

(4)

Expressions for the evaluation of the transmittance \( T \) and reflectance \( R \) of multilayer coatings are given in Sec. 42.3 under “Matrix Theory for the Analysis of Multilayer Systems.”

Providing that the incident light is not coherent, there will be no interference between the beams reflected from the two surfaces of a substrate, even when the surfaces are
plane-parallel. A summation of all the partial reflections leads to the following expression for the overall spectral transmittance $T_{\text{total}}$ of a filter:

$$T_{\text{total}} = \frac{T_1 T_2 \tau_1 \tau_2}{1 - R_1 R_2 \tau_1 \tau_2} \quad (5)$$

The reflection coefficients of uncoated interfaces can be calculated from Eq. (80), providing that the complex refractive indices of the substrate and of the medium are known. If all the materials in the filter are nonabsorbing, then

$$T_{\text{total}} = \frac{T_1 T_2}{1 - R_1 R_2} \quad (6)$$

If $R_1$ is small, an appropriate expression for $T_{\text{total}}$ is

$$T_{\text{total}} = [1 - R_1(1 - R_2)]T_2 \quad (7)$$

However, this last approximation is not valid in general; some infrared substrate materials have high reflection coefficients and in such cases Eq. (6) must be used.

**Transmission Filters in Series and Parallel**

To obtain a desired spectral transmittance, it is frequently necessary to combine several filters. One common approach is to place several filters in series (Fig. 2a).

Because of the many different partial reflections that may take place between the various surfaces, precise formulas for the resulting transmittance are complicated. Accurate calculations are best carried out using matrix methods.

To a first approximation, the resultant transmittance $T'$ of a filter system consisting of $k$ individual filters placed in series is given by

$$T' = T'_1 T'_2 T'_3 \cdots T'_k \quad (8)$$

Here $T'_i$ is the total transmittance of filter $i$. This expression is valid only if the reflectances

![FIGURE 2](image-url) Transmission filters arranged in series and in parallel. The filters can be air-spaced (a) and (c) or cemented (b).
of the individual filters are small or if the interference filters are slightly inclined to one another and the optics are arranged in such a way that the detector sees only the direct beam.

Under other circumstances, the use of this expression with interference filters can lead to serious errors. Consider two separate filters placed in series and let \( T_1', T_2', R_1', \) and \( R_2' \) correspond to the transmittances and reflectances of the two filters. If \( T_1' = T_2' = R_1' = R_2' = 0.5 \), then according to Eq. (8), the resulting transmittance will be \( T' = 0.25 \). For this simple case the precise expression can be derived from Eq. (5) and is given by

\[
T' = \frac{T_1'T_2'}{1 - R_1'R_2'}
\]

Evaluating this expression one obtains \( T' = 0.33 \). This is significantly different from the result obtained from the application of Eq. (8).

Some spectral transmittance curves cannot be easily designed by placing filters in series alone. For certain applications it is quite acceptable to place filters not only in series, but also in parallel.6 This introduces areas as additional design parameters. Thus, for example, the effective spectral transmittance \( T' \) of the filter shown in Fig. 2c would be given by

\[
T' = \left( \frac{A}{A} T_{a'} + \frac{A}{A} T_{b'} + \frac{A}{A} T_{c'} + \frac{A}{A} T_{d'} \right)
\]

where

- \( A = \) overall area of filter
- \( a, b, c, d = \) areas of the four zones
- \( T_a', T_b', T_c', T_d' = \) transmittances of four zones

The latter are given by

\[
\begin{align*}
T_{a'} & = T_1' \cdot T_2' \cdot T_3' \\
T_{b'} & = T_1' \cdot T_2' \cdot T_3' \\
T_{c'} & = T_1' \cdot T_2' \\
T_{d'} & = T_1' \cdot T_2' \cdot T_3' \cdot T_k'
\end{align*}
\]

Great care must be exercised when using such filters. Because the spectral transmittance of each zone of the filter is different, errors will result unless the incident radiation illuminates the filter uniformly. Similar care must be used when employing the filtered radiation. One way proposed to alleviate these problems is to break the filter down into a large number of small, regular elements and to reassemble it in the form of a mosaic.7

**Reflection Filters in Series**

If radiation is reflected from \( k \) different filters, the resultant reflectance \( R' \) will be given by

\[
R' = R_1' \cdot R_2' \cdot R_3' \cdots R_k'
\]

which is analogous to Eq. (8) for the resultant transmittance of filters placed in series. Many of the considerations of that section also apply here. For instance, \( R' \) will be significant only at those wavelengths at which every one of the reflectors has a significant reflectance. Metal layers (see “Metallic Reflectors” in Sec. 42.16) and thin-film interference coatings can be used exclusively or in combination.

For the sake of convenience the number of different reflecting surfaces used is
normallly restricted. The outlines of some possible reflector arrangements given in Fig. 3 are self-explanatory. The arrangement shown in Fig. 3b does not deviate or displace an incident parallel beam. The number of reflections depends in each case on the lengths of the plates and on the angle of incidence of the beam. Other arrangements are possible.

Clearly, reflection filters placed in series require more space and are more complicated to use than transmission filters. But if in a given application these shortcomings can be accepted, multiple-reflection filters offer great advantages, which stem mainly from the nature of the reflectors available for their construction (see “Multiple-reflection Filters” later in this chapter).

42.3 THEORY AND DESIGN OF OPTICAL THIN-FILM COATINGS

Design Approaches

A thin-film designer may be asked to design a multilayer coating in which the transmittance, reflectance, and/or absorptance values are specified at a number of wavelengths, angles, and polarizations of the incident light. The designer may be required to provide a coating with many other more complicated properties, including integral quantities such as CIE color coordinates, solar absorptance, or emissivity. The parameters that can be used to reach these goals are the number of layers in the multilayer, the layer thicknesses, and the refractive indices and extinction coefficients of the individual layers and of the surrounding media. Clearly, the more demanding the performance specifications, the more complex the resulting system. Many different methods have been developed for the design of multilayer coatings. For a good overview of this topic the interested reader is referred to the books by Macleod, Knittl, and by Kurman and Tikhonravov. Here only the most important methods will be mentioned.

Graphical vector methods provide the most understanding of the problem, but the necessary approximations limit them to the solution of problems in which the final reflectance is not too high. Admittance diagrams and similar chart methods do not suffer from this limitation, but they are best applied to problems in which the specifications are relatively simple. Many problems can be solved using the known properties of periodic multilayer systems. Analytical synthesis methods yield solutions to problems in which...
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Quite complex spectral transmittance or reflectance curves are specified.\textsuperscript{13,14} However, frequently solutions obtained in this way call for the use of inhomogeneous layers which are difficult to deposit, or for homogeneous layers with optical constants that are outside the range of known materials. Numerical design methods are the most flexible of all because they can be applied to problems with very complex specifications requiring a large number of layers for their solution.\textsuperscript{15,16} They are usually based on the matrix theory of optical multilayer systems and are particularly powerful for the solution of complicated spectral problems when combined with analytical methods.

**Matrix Theory for the Analysis of Multilayer Systems**

If electromagnetic radiation falls onto a structure consisting of thin films of several different materials, multiple reflections will take place within the structure. Depending on the light source and the layer thicknesses, the reflected beams may be coherent and interfere with one another. This optical interference can be used to design optical multilayer filters with widely varying spectral characteristics. In this section the basic equations for thin-film calculations are presented and some general properties of interference filters are listed. For a thorough discussion of this topic the reader is referred to the work of Macleod\textsuperscript{2,17} and Thelen.\textsuperscript{18}

Consider the thin-film system consisting of \( L \) layers shown in Fig. 4. The construction parameters comprise not only the refractive indices \( n_j \) and the thicknesses \( d_j \) of the layers \( j = 1, 2, \ldots, L \), but also the refractive indices \( n_s \) and \( n_m \) of the substrate and the incident medium. The angle of incidence \( \theta \), the wavelength \( \lambda \), and the plane of polarization of the incident radiation are the external variables of the system.

The most general method of calculating the transmittance \( T \) and the reflectance \( R \) of a multilayer from these quantities is based on a matrix formulation\textsuperscript{19,20} of the boundary conditions at the film surfaces derived from Maxwell’s equations.\textsuperscript{21}

It can be shown that the amplitude reflection \( r \) and transmission \( t \) coefficients of a multilayer coating consisting of \( L \) layers bounded by semi-infinite media are given by

\[
r = \frac{\eta_m E_m - H_m}{\eta_m E_m + H_m}
\]

\[
t = \frac{\eta_m E_m + H_m}{\eta_m E_m - H_m}
\]  

\textbf{FIGURE 4} Construction parameters of a multilayer.
and

\[
\iota = \frac{2\eta_m}{\eta_m E_m + H_m}
\]

(14)

where

\[
\begin{pmatrix}
E_m \\
H_m
\end{pmatrix} = M \begin{pmatrix}
1 \\
\eta_i
\end{pmatrix}
\]

(15)

\(E_m\) and \(H_m\) are the electric and magnetic vectors, respectively, in the incident medium, and \(M\) is a product matrix given by

\[
M = M_2 M_{2-1} \cdots M_j \cdots M_1
\]

(16)

In the preceding equation, \(M_j\) is a \(2 \times 2\) matrix which represents the \(j\)th film of the system:

\[
M_j = \begin{pmatrix}
m_{11} & m_{12} \\
m_{21} & m_{22}
\end{pmatrix} = \begin{pmatrix}
\cos \delta_j & i \frac{\sin \delta_j}{\eta_i} \\
i \eta_i \sin \delta_j & \cos \delta_j
\end{pmatrix}
\]

(17)

where

\[
\delta_j = \frac{2\pi}{\lambda} (n_j d_j \cos \theta_j)
\]

(18)

The quantity \(n_j d_j \cos \theta_j\) is the effective optical thickness of the layer \(j\) for an angle of refraction \(\theta_j\). In Eqs (13) to (17) \(\eta\) represents the effective refractive index of the medium, substrate, or layer, and is given by

\[
\eta = \begin{cases}
\frac{n}{\cos \theta} & \text{p-polarization} \\
\frac{n}{n \cos \theta} & \text{s-polarization}
\end{cases}
\]

(19)

depending on whether the incident radiation is polarized parallel (\(p\)) or perpendicular (\(s\)) to the plane of incidence. The angle \(\theta_j\) is related to the angle of incidence \(\theta\) by Snell’s law

\[
n_m \sin \theta = n_j \sin \theta_j
\]

(20)

The intensity transmittance and reflectance are

\[
T = \frac{\eta_i}{\eta_m} |\iota|^2
\]

(21)

\[
R = |\eta|^2
\]

(22)

and the phase changes on transmission and reflection, \(\varepsilon_T\) and \(\varepsilon_R\) are given by

\[
\varepsilon_T = \text{arg} \, \iota
\]

(23)

\[
\varepsilon_R = \text{arg} \, \eta
\]

(24)

If the materials in a multilayer are all nonabsorbing, then \(T + R = 1\). Should one or
more materials absorb, then in the preceding equations the refractive indices of these materials must be replaced by their complex refractive indices \( \tilde{n} \), defined by

\[
\tilde{n} = n - ik
\]

where \( k \) is the extinction coefficient of the material. Even though all the elements of the layer matrix for such a material are now complex, its determinant will still be unity. The absorptance of the multilayer is then calculated from

\[
A = 1 - T - R
\]

Certain important general conclusions about the properties of multilayer filters can be drawn from these equations.

1. The properties of thin-film systems vary with angle of incidence [Eqs. (18), (19)]. For some applications this is the major disadvantage of interference filters compared to absorption filters.

2. This variation depends on the polarization of the incident radiation [Eq. (19)]. The following equations define \( T \) and \( R \) for obliquely incident randomly polarized radiation:

\[
T = \frac{1}{2}(T_p + T_s) \tag{27}
\]

\[
R = \frac{1}{2}(R_p + R_s) \tag{28}
\]

The dependence of \( T \) and \( R \) on polarization has been used for the design of polarizers (see Sec. 42.11 and Chap. 3, Vol. II of this Handbook). However, like the angular variation, it is a disadvantage for most other applications. Many researchers have investigated ways of reducing these effects.18, 22–25

3. Transmittance curves of nonabsorbing multilayers composed of layers whose optical thicknesses are all multiples of \( \lambda/4 \) show symmetry about \( \lambda_o \) when plotted on a relative wavenumber scale \( \lambda_o/\lambda \) [Eqs. (17) and (18)].

4. A proportional change of all the thicknesses of a nonabsorbing multilayer results merely in a displacement of the transmittance curve on a wavenumber scale [Eq. (18)]. Thus a thin-film design can be utilized in any part of the spectrum subject only to the limitations imposed by the dispersion of the optical constants of the materials used.

5. The reflectance and absorptance of a filter containing absorbing layers will depend, in general, on which side of the filter the radiation is incident (Fig. 5). However, the transmittance does not depend on the direction of the incident light.

---

**FIGURE 5** Spectral characteristics of a thin chromium alloy film on glass. Curve 1: transmittance; curves 2 and 3: reflectance from the glass and air sides, respectively. (After Liberty Mirror.)
An important parameter is the electric field amplitude squared, $|E|^2$. The susceptibility of a multilayer to high-power laser damage is proportional to the highest value of this quantity within the multilayer. One way to evaluate $E$ is to subdivide each layer of the system into $L_{\text{sub}}$ sublayers (Fig. 6) and to evaluate the partial matrix products $P_j$,

$$P_j = \left( \begin{array}{c|c} p_{11} & p_{12} \\ \hline p_{21} & p_{22} \end{array} \right) = \prod_{k=1}^{j} (S_k)$$

(29)

for all $L_{\text{sub}}$ sublayers, where $S_k$ is the matrix of the $k$th sublayer. Then, the electric field amplitude at a point $j$ in the multilayer is given in terms of the elements of the total and partial product matrices by the following expression:

$$E_j = \frac{4\left[p_{11}^j + (n_s p_{22})^2\right]}{[m_{11} + \left(n, m_{22}/n_m\right)^2] + \left[(m_{21}/n_m) + n, m_{12}\right]^2}$$

(30)
The analysis of optical thin-film systems [with computer programs based on Eqs. (13) to (30)] is relatively simple. The design of filters with any but the simplest spectral characteristics remains a complicated problem and one of the methods listed under “Design Approaches” must be used.

42.4 THIN-FILM MANUFACTURING CONSIDERATIONS

The optical, mechanical, and environmental properties of multilayer coatings depend on the materials used, the deposition process, and on the surface quality of the substrate. There are many different methods for the deposition of thin films. Some of the more common processes are reviewed below. The deposition methods and process parameters used affect the microstructure of the resulting layers. The films can be dense with an amorphous or a microcrystalline structure, or they may exhibit a columnar growth with considerable voids. The optical constants clearly depend on this microstructure and films of the same materials may sometimes have very different properties, depending on how they were deposited (see, for example, Ref. 30). The individual films in a multilayer may be under tensile or compressive stress and, unless materials and film thicknesses are selected to compensate for these stresses, the overall stress may be large enough to distort the substrate or cause the multilayer to break up. The mechanical properties of multilayer coatings also critically depend on the microstructure of the films. An excellent discussion of the effects of microstructure on the various properties of optical coatings has been given by Macleod.

Optical Coating Materials

Many different materials have been used in the past for the construction of optical multilayer coatings. Some of the compounds used for the deposition of nonabsorbing layers of the ultraviolet, visible, and infrared parts of the spectrum are cryolite (1.35), LiF (1.37), MgF₂ (1.39), ThF₄ (1.52), CeF₃ (1.62), PbF₂ (1.73), ZnS (2.30), ZnSe (2.55), Si (3.5), Ge (4.20), Te (4.80), PbTe (5.50), SiO₂ (1.48), Al₂O₃ (1.60), MgO (1.72), Y₂O₃ (1.82), ZrO₂ (1.86), SiO (1.95), HfO₂ (1.98), ZrO₂ (2.10), CeO₂ (2.20), Nb₂O₅ (2.20), Ta₂O₅ (2.10), and TiO₂ (2.45). The numbers in parentheses represent the approximate refractive indices at the midpoints of the material transparency range. Some of the metals used in the same wavelength range for the deposition of reflecting or absorbing layers are Ag (0.12–1.34), Al (1.02–6.85), Au (0.31–2.88), Cu (0.83–2.60), Ni (1.80–3.33), Cr (3.18–4.41), Inconel (2.94–2.92), and Rb (2.00–5.11). The complex refractive indices of the metals given in the brackets correspond to a wavelength of 0.56 μm. These values are approximate and are intended as a rough guide only. More extensive listings of coating materials are given, for example, by Macleod and Costich.

As already mentioned, the properties of multilayers depend on the materials used for their construction. For example, layers made of oxides are, as a rule, harder than those made of fluorides, sulphides, or semiconductors. They are therefore preferred for use on exposed surfaces. Semiconductor materials should be avoided in filters that are to be used over a wide range of temperatures because their optical constants can change significantly. Some metals are soft and easily damaged while others tarnish when exposed to the atmosphere. Such coatings require further protective coatings, or should be cemented between two transparent plates. Other materials require the precoating with adhesion
layers to ensure a good bond to the substrate. For example, frequently an Ni adhesive layer is deposited onto glass before coating with Au.

**Evaporation**

*Conventional (nonreactive) or reactive evaporation* from resistance, induction, or electron beam gun sources is a low-energy process (~0.1 eV) and the resulting films frequently have a porous structure. The porosity may vary with the material, the substrate temperature, the residual pressure in the deposition chamber, the deposition rate, and angle of incidence of the vapor on the substrate. Values of porosities ranging from 0 to 40 percent have been observed. On exposure to the atmosphere, some of the voids in the film may absorb water vapor. This increases the effective refractive index of the films and results in a shift of the spectral features of the multilayer towards longer wavelengths ("ageing"). This shift is partially reversible—by placing the filter in an inert atmosphere or in a vacuum, or on heating, some of the adsorbed water vapor can be removed. Unless it has been allowed for at the design stage, such ageing can render some filters useless.

The microstructure of the films can be significantly affected by bombarding the substrate during deposition with energetic ions from an auxiliary ion beam source. The additional energy (~50 to 100 eV) results in denser films. Hence, coatings produced by *ion-assisted deposition* have higher refractive indices and exhibit less ageing on exposure to the atmosphere.

The *ion plating* process can result in even denser coatings. In this high deposition rate process the starting material must be a good conductor and is usually a metal. Argon and a reactive gas species are introduced into the chamber and, together with the evaporant, are ionized. The ions are then accelerated to the substrate with energies of the order of 10 to 50 eV. Transparent films with near-bulklike densities and low temperature variation of refractive index can be obtained by this process. For most materials, the layers are glasslike and the interfaces remain smooth. This results in a lower scatter.

Conventionally evaporated thin films can be under compressive or tensile stresses. If not controlled, these stresses can distort the substrate or cause the multilayer to break up. The magnitude of the stresses depends on the material and on the deposition conditions. It is possible to select the materials and process parameters so that the stresses of the various layers counteract each other. In contrast, almost all ion plated layers are under compressive stress. It is therefore more difficult to produce stress-compensated multilayers by this process.

**Sputtering**

*Reactive or nonreactive dc or RF magnetron sputtering* is also used to deposit optical multilayer coatings. Many variants of this process exist. Most are significantly slower than evaporation and the targets can be quite expensive. Filters produced by dc or RF sputtering may therefore also be more expensive. However, the process is stable, provides excellent control over the thicknesses of the layers, and can be readily scaled to provide uniform coatings over large areas. Both metal and metal oxide layers can be produced. Sputtering is an energetic process and results in dense, bulklike layers which exhibit virtually no ageing.

In *ion-beam sputtering*, an energetic beam of inert ions is aimed at a target made of the material that is to be deposited. Atoms or clusters of atoms of the material are dislodged from the target and land on the substrate with a high energy. This is the slowest physical vapor thin-film deposition method described here and it cannot be readily scaled for the coating of large components. However, it yields the highest quality coatings. Many of the
high-reflectance coatings for laser gyros, in which no significant losses can be tolerated, are produced in this way.

**Deposition from Solutions**

In this procedure the substrate is either dipped in an organo-metallic solution and withdrawn at a very steady rate from it, or the solution is applied from a pipette onto a spinning substrate. The substrate is then placed in an oven to drive off the solvent. The thickness of the film depends on the concentration of the solvent and on the rate of withdrawal or spinning. Other factors which influence the process are temperature and humidity, as well as the freshness of the solution. Although it yields quite porous films, this method is of interest because many of the layers produced in this way have a high laser damage threshold. The process has also been adapted for the coating of quite large area substrates with multilayer antireflection coatings for picture frame glass and for display windows.

**Thickness Control during Deposition**

The performance of many optical multilayers depends critically on the thicknesses of the individual layers. The control of the layer thicknesses during their formation is therefore very important. Many different methods exist for the monitoring of layer thicknesses. For very steady deposition processes, such as sputtering, simple timing can give good results. However, the most common techniques used are quartz crystal and optical monitoring. The former is very sensitive and can be used for thin and thick films, as well as transparent and opaque films. However, it is an indirect method and requires careful calibration. This is usually not a problem whenever layers of established coating materials are formed using a standard geometry and deposition conditions. Optical monitoring can be performed directly on the substrate, or indirectly on a witness glass. The quantities measured are usually $T$, $R$, or the ellipsometric parameters. One advantage of direct optical monitoring is that the parameters measured are usually closely related to the required performance. Furthermore, with optical monitoring, a real-time error determination and compensation is possible after the deposition of each layer, through the reoptimization of the remaining layers of the system. With this method even quite complicated multilayer structures can be manufactured.

**42.5 Measurements on Optical Coatings**

**Optical Properties**

*Transmission, Reflection, and Absorption.* The most commonly used instrument for the measurement of the optical performance of thin-film coatings is the spectrophotometer. The wavelength dispersion of commercial instruments for the 0.185- to 8.00-μm spectral range is usually provided by prisms or by ruled or holographic gratings. Grazing incidence gratings, crystals, or multilayer coatings are used in the soft x-ray and extreme ultraviolet (XUV) spectral regions. Fourier transform spectrometers are capable of measurements from about 2 to 500 μm. A variety of attachments is available for the measurement of specular and diffuse reflectance. Absolute measurements of $T$ and $R$ that are accurate to within ±0.1 percent are difficult even in the visible part of the electromagnetic spectrum.
Very small absorptions of single layers are normally measured with calorimeters\(^40\). The losses of high-performance laser reflectors are obtained from measurements of the decay times of Fabry-Perot interferometer cavities formed from these mirrors\(^31\).

Roughness of the substrate and irregularities occurring within individual films and the layer interfaces give rise to light scattering in all directions\(^42–44\). For many applications, it is important to minimize this scatter. Special instruments, called scatterometers, are used to measure the angular variation of the light scatter. Such data provides information about the substrate and multilayer.

The transmittance, reflectance, and absorptance of some optical coatings are affected by exposure to atomic oxygen and by electron, proton, and ultraviolet irradiation. They also depend critically on the cleanliness of the components measured. In space, contamination of optical components can also take place\(^45,46\).

**Optical Constants.** A reliable knowledge of the optical constants of all the materials used in the construction of optical multilayer coatings is essential. There exist many different methods for their determination\(^47\). These include methods that are based on refractometry, photometric and spectrophotometric measurements of \(R\) and/or \(T\), polarimetry, single-wavelength or spectroscopic ellipsometry, various interferometric methods, attenuated total reflection, or on a combination of two or more of these methods. Excellent monographs on the various methods will be found in Palik's *Handbook of Optical Constants of Solids*\(^48,49\). Some are suitable for measurements on bulk materials and the results are valid only for films produced by the more energetic deposition processes described here. The optical constants of porous films must be measured directly. They will depend on the deposition parameters and on the layer thickness, and may differ significantly from those of bulk materials\(^50\). Special methods have to be used for the determination of the optical constants in the x-ray, XUV, and submillimeter regions. The accurate measurement of very small, residual extinction coefficients of transparent coating materials is difficult. Generally, it involves the use of laser calorimetry or the use of the film as a spacer layer in a bandpass filter. It is also very important to be able to measure the thickness of the film independently.

**Laser Damage.** A measure of the ability of an optical component to withstand high laser irradiations is its laser-induced damage threshold (LIDT). There are several ways of defining this quantity. One frequently used definition is based on a survival plot of the percentage of components that are damaged when they are exposed to different laser fluences. The value of the fluence corresponding to the intersection of a mean curve through the experimental points with the ordinate is nominally defined to be the LIDT. It is thus the maximum fluence at which no damage is expected to the component. The slope of the survival curve is also usually reported. Methods for the determination of the LIDT differ in the way in which the damage to the component is observed. This may, for example, be the observation of a catastrophic failure with a Nomarski microscope, or the detection of a specified change in scatter, or the relaxation of the material as seen with a photothermal deflection method, or an operational criterion, where, within specified limits, the component can no longer perform the function for which it was designed.

Absorption is the main cause for laser damage. The incident radiation that is absorbed in the optical component is usually converted into heat in the damage process. If the thermal conductance of the optical component is too low, the temperature of the local absorbing spot on the element will rise to a value at which damage ensues. The damage will therefore depend on the thermal conduction of the materials of which the element is made. For example, some high-reflectivity mirrors consist of coatings on Si, SiC, W, or Cu substrates that are water-cooled during use to increase heat removal.

Thermal conductivities of thin films are several orders of magnitude smaller than those of the corresponding bulk materials. (An exception to this are some fluoride layers.) This compounds the problem. To increase the LIDT, the deposition methods are optimized to
obtain homogeneous thin films of low absorption and with more bulklike thermal conductivities.

Absorption damage usually initiates at defects and other imperfections. In the case of the substrate, it may occur at or below the surface, even when the substrate material is nonabsorbing. It is very important to avoid materials with color centers, subsurface damage, and inclusions. In point of fact, in most pulsed laser-induced damage to thin films, damage occurs at discrete locations such as nodules. The substrate surface must be very smooth and devoid of scratches, digs, and pores, otherwise polishing compounds and other contaminants can be trapped. It is imperative that the substrate be as perfectly clean as possible prior to coating. Electric fields associated with these imperfections can increase the absorption by an amount which is proportional to the refractive index of the material.

The coating materials used for the deposition of high LIDT multilayers must be very pure, with absorption edges far away from the wavelength of interest and, for many pulsed irradiations, from the harmonics as well. As a rule, materials in thin-film form have extinction coefficients that are orders of magnitude larger than those of the corresponding bulk materials. Currently, the processes used to produce high LIDT coatings include ion-assisted deposition, ion-beam sputtering, sol-gel deposition, and electron beam gun evaporation. To reduce the effects of the residual surface roughness, the thicknesses of the layers are often adjusted to shift the peaks of the electric field away from the layer boundaries where impurities tend to concentrate.

The form that the damage takes depends to a large extent on the materials. Pitting of the coatings is probably due to the explosion of nodules. Delamination may be due to poor adhesion of the layers to the substrate, to undue stresses in the films, and/or to a poor match between the expansion coefficients of the layers and the substrate.

The LIDT also depends on the laser pulse duration. For very short pulses (<10 ns) thermal conductance does not play as major a role in the process. For longer pulses, the LIDT is frequently seen to be proportional to the square root of the pulse duration, indicating a mechanism that depends mainly on diffusion. For high repetition rates the LIDT depends somewhat on the repetition rate. To achieve a long life (>10⁶ pulses) in an industrial environment, the laser should be operated at a fraction (say, 1/4 or 1/10) of the nominal LIDT. In CW lasers it is not the LIDT, but the average power-handling capability (i.e., the energy) that is of essence. Long before damage takes place, the heating can cause a distortion of the surface which, in turn, can result in a loss of power, in mode and focusing problems, and even in material fracture.

The LIDT of an element can be increased through preconditioning. This consists of a special irradiation protocol applied to the component prior to its first use. On the other hand, some materials exhibit accumulation effects, e.g., through color centering, which reduce the LIDT on repeated exposure.

The development of laser coatings with high LIDT is so important that conferences have been held in Boulder, Colorado on this topic every year since 1969. For more detail the reader is referred to the proceedings of these conferences, as well as to a draft international standard on this topic. An increasing number of thin-film vendors include in their catalogues LIDT information on all or some of their products. Independent LIDT test services are now provided by several commercial companies and publicly funded institutions.

### Mechanical Properties

Optical multilayer coatings are frequently required to operate under severe mechanical and environmental conditions. A number of standards deal with the substrate and coating quality (MIL-0-13830B), the adhesion of coatings (MIL-M-13508C; MIL C 48497), their abrasion resistance (MIL-C-675A, MIL C 675C, MIL-C-14806-A, MIL C 48497), hardness (MIL-M-13508C), and resistance to humidity (MIL-C-675A, MIL-C-14806-A, MIL-810-C,
MIL C 48497), salt solution (MIL-C-675-A), and salt spray (MIL-M-13508-C, MIL-C-14806-A). Most of these standards are reprinted in Rancourt’s book. Depending on the application, multilayer coatings may be required to meet one or more of these standards. An overview of the subject of stresses and hardness of thin films on a substrate has been recently published by Baker and Nix.

**Analytical Analysis Methods**

In addition to optical and mechanical measurements, multilayer coatings can be subjected to a number of analytical measurements. These include Auger electron spectroscopy, energy dispersive x-ray analysis, Rutherford backscattering, secondary ion mass spectrometry, transmission electron microscopy, and x-ray photoelectron spectroscopy. Some of these tests are destructive. However, when a multilayer coating is subjected to these tests, they yield fairly accurate information about the number of layers in a system, and on the composition, thickness, and structure of the individual layers.

### 42.6 ANTIREFLECTION COATINGS

**Effect of Surface Reflections on Performance of Optical Systems**

The reflectance of an interface between two nonabsorbing media of refractive indices $n_1$ and $n_2$ is given by

$$R = \left( \frac{n_1 - n_2}{n_1 + n_2} \right)^2$$

(31)

An expression for the total transmittance $T_o$ of a nonabsorbing plane-parallel plate that takes into account the effect of multiple internal reflections within the plate can be obtained from Eq. (9):

$$T_o = \frac{1 - R}{1 + R}$$

(32)

Of this light only a fraction $(1 - R)^2$ passes through the plate without undergoing any reflections.

An expression for the transmission of a number of such plates placed in series is of interest. It helps to explain the effect of multiple reflections between the various plates on the performance of devices such as triple glazings and photographic objectives. It can be shown that the total transmittance $T_{\text{total}}$ of $m$ plates placed in series is given by

$$T_{\text{total}} = \frac{T_o}{m - T_o(m - 1)}$$

(33)

The amount of light transmitted directly $T_{\text{direct}}$, is

$$T_{\text{direct}} = (1 - R)^m$$

(34)
The light $T_{\text{stray}}$ that undergoes multiple reflections before transmission is responsible for spurious images and stray light in the image plane. It is given by

$$T_{\text{stray}} = T_{\text{total}} - T_{\text{direct}}$$

(35)

The variation of $T_{\text{direct}}$ and $T_{\text{stray}}$ with $R$, for several values of $m$, is shown in Fig. 7. The refractive indices of the plate material plotted against the upper $x$ axis assume that the plates are in air.

It will be seen that even for a relatively small number of low-refractive-index plates the ratio $T_{\text{stray}}/T_{\text{direct}}$ becomes significant. This means that, in an image-forming system under unfavorable conditions, the stray light can completely obscure the image. Second, even in nonimaging optical systems, the loss of light $(1 - T_{\text{direct}} - T_{\text{stray}})$ can become quite prohibitive.

Both these problems can be overcome by reducing the surface reflection through the application of suitable antireflection coatings to the plate boundaries. Since antireflection coatings with zero reflectance across the whole spectrum cannot be constructed, the spectral reflectance $R(\lambda)$ of antireflection coatings is usually chosen to minimize the integral

$$\int R(\lambda)I(\lambda)S(\lambda) d\lambda$$

(36)

where $I(\lambda)$ and $S(\lambda)$ are the spectral-intensity distribution of the incident radiation and the spectral sensitivity of the detector, respectively. A low reflectance is thus needed only in the spectral region in which $I(\lambda)S(\lambda)$ is significant. It should be emphasized here that in addition to this requirement, for most applications, antireflection coatings must be mechanically very tough, withstand drastic climatic and thermal variations, and stand up to the usual lens-cleaning procedures. Some examples of improvements in the performance of image-forming and non-image-forming optical systems obtained through the use of antireflection coatings are given by Mussett and by Faber et al.

Antireflection coatings can be based on homogeneous layers or on inhomogeneous
coatings. One can further classify them into single-layer, digital, or structured, and homogeneous multilayer or complex inhomogeneous layer coatings (Fig. 8a to d). Because of their industrial importance, antireflection coatings for the visible and infrared spectral regions have been the subject of much research and development. Two books have been written on this topic\textsuperscript{57,58} and there exists a very extensive literature in scientific and technical journals. For a review of this literature and for a systematic discussion of antireflection coatings, the reader is referred to the excellent review articles by Cox and Hass\textsuperscript{59} and by Mussett and Thelen.\textsuperscript{55} In this section, only a brief summary will be given of the results obtained thus far, intended to aid in the selection of antireflection coatings for particular applications. The calculated data is presented on a logarithmic scale. The relative wave-number scale facilitates the calculations of the width of the effective region of coating in different parts of the spectrum.

**Antireflection Coatings Made of Homogeneous Layers**

The single homogeneous-layer antireflection coating (Fig. 8a) was the first antireflection coating and perhaps is still the most widely used. Theoretically, it should be possible to obtain a zero reflectance at one wavelength with single dense films. However, because of a lack of suitable low-index coating materials, this cannot be realized in practice for substrates with refractive indices less than about 1.9. Nevertheless, even with the available
materials, a very useful reduction in reflection in a broad spectral region is obtained for all common glass types, the reflectance never rising above that of the uncoated surface (Fig. 9, curve 8). With the sol-gel method it is possible to produce homogeneous porous oxide and fluoride films with very low refractive indices.60–63 Such films have excellent optical characteristics (Fig. 9) and have laser damage thresholds that are considerably higher than those produced by conventional means. However, these gains are at the expense of mechanical strength and long-term stability. Low-refractive-index coating materials can also be simulated by the deposition or etching of subwavelength structures (Fig. 8b, e). The effective refractive index depends on the volume fraction occupied by the structures.64

If more than one layer is used, all the degrees of freedom could be used to either (1) obtain a more complete antireflection in one particular spectral region; (2) increase the width of the spectral region over which the reflectance is generally low; or (3) obtain a coating in which the low reflectance is very uniform across the spectrum.65,66 Vlasov has shown with the aid of a diagram of the type shown in Fig. 10 that even for a two-layer antireflection coating there exists a large number of refractive-index combinations which will yield zero reflectance at one wavelength.57 As the number of layers and the overall thickness of the antireflection coating increases, it becomes possible to find solutions for a particular problem which not only fully meet the most important above desirata and almost satisfy the others, but are also based on the use of the mechanically most satisfactory coating materials.

The conditions that are satisfied by the refractive indexes and thicknesses of various types of antireflection coatings are given in Table 1. In a few cases where they are very complicated, reference is made to a paper in which they are set out in full. The calculated transmittance curves of antireflection-coated surfaces of glass (Fig. 11a to e), quartz (Fig. 11d), germanium (Fig. 11e and f), silicon (Fig. 11g), and other infrared materials (Fig. 11h) utilize refractive indices that for the most part correspond to real coating materials, and hence the curves represent realistic solutions rather than the theoretically best possible ones. The actual refractive indices used in the calculations (and optically thicknesses where they do not correspond to a multiple of λ/4) are given in Table 1.

Figure 11a shows the performance of several antireflection coatings on glass for
FIGURE 10 Refractive-index combinations (shaded areas) of two-layer antireflection coatings with which a zero reflectance at one wavelength can be achieved.

applications which require the highest possible efficiency for a limited wavelength region. Of these, the coating 2.1 has probably found the greatest acceptance. A typical measured-performance curve for such a coating is shown in Fig. 12. Antireflection coatings with a low reflectance in a broad spectral region are shown in Fig. 11b and 13. Coatings of the type 3.4 probably find the widest application in practice. The performance of commercial coatings for quartz substrates is shown in Fig. 14.

Solutions listed in Table 1 presume that coating materials with the required refractive indices exist. This is frequently not the case and the thin-film designer must seek solutions based on available coating materials. For example, Furman\(^6\) gives a series of practical two-, three-, and four-layer solutions for ten different values of the substrate index ranging between 1.5 and 4.0, while Stolov\(^8\) provides seven-layer solutions for ten values of \(n_s\), 1.46 \(\leq n_s \leq 1.82\).

Willey has given a useful empirical expression which relates the average reflectance \(R_{\text{avg}}\) in the visible and near-infrared spectral region \(\lambda_{\text{min}} \leq \lambda \leq \lambda_{\text{max}}\) to the overall optical thickness \(\Sigma(nd)\) of the coating of an antireflection coating composed of layers of refractive indices \(n_M, n_H\) and a single, outermost layer of index \(n_L\):\(^6\)

\[
R_{\text{avg}} = 0.01 \cdot \left[\frac{\max(\lambda)}{\min(\lambda)} (n_L - 1)\right]^{0.63} \left(\frac{\max(\lambda)}{\Sigma(nd)}\right)^{0.63} \left[(1.2 - \Delta n)^2 + 0.42\right]
\]

The preceding expression is valid for the following parameter values:

\[
1.5 \leq \frac{\lambda_{\text{max}}}{\lambda_{\text{min}}} \leq 3.0 \quad 1.17 \leq n_L \leq 1.46 \quad 1.38 \leq n_M, n_H \leq 2.58
\]

\[
1.0 \leq \frac{\Sigma(nd)}{\lambda_{\text{max}}} \leq 3.0 \quad 0.4 \leq \Delta n = n_H - n_M \leq 1.2
\]

A method for the design of optimum or near-optimum two-material antireflection coatings for a given substrate, coating materials, and overall thickness has recently been described.\(^7\)

Since for some applications the color introduced into an optical system by antireflection coatings is of paramount importance, it has been the subject of many studies.\(^7\) One way
### Table 1: Some Antireflection Coatings

(The incident medium in all cases is air.)

<table>
<thead>
<tr>
<th>Type</th>
<th>Conditions or reference</th>
<th>Substrate material</th>
<th>$n_a$</th>
<th>$n_1$ ($n_1 d_1/\lambda$)</th>
<th>$n_2$ ($n_2 d_2/\lambda$)</th>
<th>$n_3$ ($n_3 d_3/\lambda$)</th>
<th>$n_4$ ($n_4 d_4/\lambda$)</th>
<th>$n_m$</th>
</tr>
</thead>
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<tr>
<td>1.1</td>
<td></td>
<td>Glass</td>
<td>1.51</td>
<td>1.38</td>
<td></td>
<td></td>
<td></td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td>InSb</td>
<td>2.20</td>
<td>1.59</td>
<td></td>
<td></td>
<td></td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td>InAs</td>
<td>4.00</td>
<td>2.20</td>
<td></td>
<td></td>
<td></td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Ge</td>
<td>4.10</td>
<td>2.20</td>
<td></td>
<td></td>
<td></td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Si</td>
<td>3.50</td>
<td>1.85</td>
<td></td>
<td></td>
<td></td>
<td>1.00</td>
</tr>
<tr>
<td>2.1</td>
<td></td>
<td>Glass</td>
<td>1.51</td>
<td>2.30 (0.0524)</td>
<td>1.38 (0.3250)</td>
<td></td>
<td></td>
<td>1.00</td>
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<tr>
<td></td>
<td></td>
<td>Quartz</td>
<td>1.48</td>
<td>2.09 (0.0947)</td>
<td>1.48 (0.3255)</td>
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<td></td>
<td>1.00</td>
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<tr>
<td></td>
<td></td>
<td>Ge</td>
<td>4.10</td>
<td>1.35 (0.0951)</td>
<td></td>
<td></td>
<td></td>
<td>1.00</td>
</tr>
<tr>
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<td></td>
<td>Glass</td>
<td>1.51</td>
<td>1.70</td>
<td>1.38</td>
<td></td>
<td></td>
<td>1.00</td>
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<td></td>
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<td>1.70</td>
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<td>1.35</td>
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<td>1.47</td>
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<td>Si</td>
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<td>2.56</td>
<td>1.86</td>
<td>1.38</td>
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<td></td>
<td>Glass</td>
<td>1.51</td>
<td>1.65</td>
<td>2.10</td>
<td>1.38</td>
<td></td>
<td>1.00</td>
</tr>
<tr>
<td>3.5</td>
<td></td>
<td>Quartz</td>
<td>1.48</td>
<td>1.65</td>
<td>2.10</td>
<td>1.38</td>
<td></td>
<td>1.00</td>
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<tr>
<td>3.6</td>
<td>(see Thetford, Ref. 530.)</td>
<td>Glass</td>
<td>1.52</td>
<td>1.80 (0.1799)</td>
<td>2.20 (0.4005)</td>
<td>1.38 (0.2402)</td>
<td></td>
<td>1.00</td>
</tr>
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TABLE 1 Some Antireflection Coatings (Continued)

(The incident medium in all cases is air.)

<table>
<thead>
<tr>
<th>Type</th>
<th>Conditions or reference</th>
<th>Substrate material</th>
<th>( n_s )</th>
<th>( n_1 ) ((n_1d_1/\lambda))</th>
<th>( n_2 ) ((n_2d_2/\lambda))</th>
<th>( n_3 ) ((n_3d_3/\lambda))</th>
<th>( n_4 ) ((n_4d_4/\lambda))</th>
<th>( n_m )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.7</td>
<td>( n_1d_1 = n_2d_2 = n_3d_3 = \frac{\lambda}{4} ) (see Kard, Ref. 529.)</td>
<td>Glass</td>
<td>1.55</td>
<td>1.53</td>
<td>1.454</td>
<td>1.32</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>4.1</td>
<td>( n_1n_4 = n_2n_3n_1 ) ( n_1d_1 = n_2d_2 = \frac{1}{2}n_3d_3 = n_4d_4 = \frac{\lambda}{4} )</td>
<td>Glass</td>
<td>1.51</td>
<td>1.38</td>
<td>1.548</td>
<td>2.35</td>
<td>1.38</td>
<td>1.00</td>
</tr>
<tr>
<td>4.2</td>
<td>( n_1n_4 = n_2n_3 = n_wn_i ) ( n_1d_1 = n_2d_2 = n_3d_3 = n_4d_4 = \frac{\lambda}{4} )</td>
<td>Ge</td>
<td>4.0</td>
<td>2.96</td>
<td>2.20</td>
<td>1.82</td>
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<td>1.00</td>
</tr>
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<td>4.3</td>
<td>( n_1d_1 = n_2d_2 = n_3d_3 = \frac{\lambda}{4} ) (see Kard, Ref. 529.)</td>
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<td>1.55</td>
<td>1.846</td>
<td>2.289</td>
<td>2.014</td>
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<tr>
<td>4.4</td>
<td>(Design derived from Kard, Ref. 529.)</td>
<td>Glass</td>
<td>1.55</td>
<td>1.656 ((0.2417))</td>
<td>1.888 ((0.2463))</td>
<td>1.832 ((0.2390))</td>
<td>1.38 ((0.2424))</td>
<td>1.00</td>
</tr>
<tr>
<td>10.1</td>
<td>( n_1d_1 = n_2 - (11 - l) \frac{n_s - n_b}{10} ) ( l = 1, 2, \ldots, 10 ) ( n_1d_1 = n_2d_2 = \cdots = n ldap_10 = \frac{\lambda}{4} ) [see Berning, Ref. 87.]</td>
<td>Ge</td>
<td>4.00</td>
<td>( n_i = 3.735 )</td>
<td>( \cdots )</td>
<td>( n_b = 3.735 )</td>
<td>( n_{10} = 1.35 )</td>
<td>1.00</td>
</tr>
</tbody>
</table>

To avoid the problem is to utilize coatings that are particularly achromatic (Fig. 11c). Thus, for example, 50 surfaces coated with antireflection coating 4.4 would have a transmittance of 78 ± 3 percent across the whole visible spectrum. Nevertheless, of the coatings shown in Fig. 11c, only the single-layer antireflection coating is being used extensively. Antireflection coatings can also be used to correct the residual color of lens systems.75

Often transmittance rather than reflectance measurements are performed to evaluate antireflection coatings. In general, it is incorrect to assume that \( T = 1 - R \). For instance, the transmission of antireflection-coated infrared materials depends not only on the efficiency of the antireflection coating but also on the thickness and temperature of the material. This is because of the finite scatter and absorption in such materials and because of the dependence, in some cases, of the latter on temperature. The measured spectral transmittances of three common antireflection-coated infrared materials at room temperature are shown in Fig. 15. Curves for other materials are given by Cox and Hass.76

**Inhomogeneous and Structured Antireflection Coatings**

The interface between two media with refractive indices \( n_1 \) and \( n_2 \) can be antireflection-coated over a very broad spectral region by the application of a transition layer with an index that changes continuously from \( n_1 \) to \( n_2 \) (Fig. 8d). Many different refractive-index profiles have been investigated in the past.77–80 Although some of these profiles are more...
Figure 11 Calculated performance of various antireflection coatings. The numbers identifying the individual curves refer to Table 1. (a) High-efficiency antireflection coatings for glass; (b) broadband antireflection coatings for glass; (c) highly achromatic antireflection coatings for glass; (d) antireflection coatings for quartz.

effective than others, all reduce the reflectance to a fraction of a percent over the spectral region in which the coatings are transparent and do not scatter excessively, and for which the optical thickness of the layer is at least one half-wavelength (Fig. 16a). A further advantage of inhomogeneous antireflection coatings is that they are not sensitive to the angle of incidence. Processes used for the production of inhomogeneous antireflection coatings include various additive, subtractive, additive/subtractive, and replication methods. Excellent reviews of this topic exist.

In the additive method, relatively dense inhomogeneous layers of varying compositions of two or more compounds are formed on the substrate by physical or chemical deposition processes. Such coatings are mechanically more durable than any other described in this section. However, because of a lack of coating materials with refractive indices lower than about 1.35, solid inhomogeneous layers are not very suitable for the antireflection coating of air-glass interfaces. The several different inhomogeneous antireflection coatings of this type described in the past do not offer any special advantages over those composed of homogeneous layers. The situation is different in the case of high-index materials (Fig. 17). An even lower reflectance can be achieved by ending the inhomogeneous layer when its index is equal to the square of that of the lowest-index coating material available. It is then possible to complete the coating by depositing an additional homogeneous quarter-wave-thick layer of that material (Fig. 17a).

A dense inhomogeneous layer can be approximated by a series of homogeneous layers of gradually decreasing refractive indices (curve 10.1, Fig. 11f). Such layers can be prepared by evaporating a series of appropriate mixtures of two coating materials or, without mixing, by using the Herpin equivalent-index concept to simulate intermediate
refractive indices. An even more practical solution is to replace the inhomogeneous layer by a series of thin homogeneous layers of two materials only (Fig. 18).\(^\text{30}\)

In another additive process, a refractive index variation down to a value of 1.0 is achieved by depositing onto the substrate microspheres of transparent oxides or fluorides which form pyramidlike clumps (Fig. 8e).\(^\text{91}\) If losses due to scattering are to be low, the average lateral size of the features of this structure must be a small fraction of the shortest wavelength for which the coating is to be effective. A reflectance of 0.3 percent can be achieved, but the films are fragile.

The subtractive methods are attractive because they do not require expensive deposition equipment. The surface to be antireflection-coated is leached and/or etched to

![Figure 11](image1.png)

**Figure 11** (Continued) (e) antireflection coatings for germanium; (f) broadband antireflection coatings for germanium, (g) antireflection coatings for silicon; (h) single-layer antireflection coatings for Irtran II, InAs, and InSb.

![Figure 12](image2.png)

**Figure 12** Reflectance of a high-efficiency antireflection coating on glass. (After Costich.\(^\text{40}\))
form a porous transition layer in which the index varies with thickness. Not all optical materials can be treated in this way. However, special phase-separable glasses have been developed that lend themselves well to this process.\textsuperscript{92–95} Fairly durable antireflection coatings with a reflectance of less than 0.5 percent for the 0.35- to 2.5-\m\ spectral region have been produced in this way (Fig. 19a). Some other materials, such as Lexan, Mylar, and CR-39 plastic, require an ion implantation pretreatment before the etching can be applied.\textsuperscript{78,96}

In the technologically important additive/subtractive method, a single glasslike film is first deposited by a sol-gel process onto the surface that is to be antireflection coated. The composition of the film is such that, after phase separation, it can be readily leached and/or etched to form a porous microstructure with a controlled refractive index gradient.\textsuperscript{97,98} This eliminates the need for the use of expensive phase separation glass components. Such coatings have reflectances as low as 0.13 percent and a laser damage threshold that is four times higher than that of coatings produced by conventional physical vapor deposition techniques (Fig. 19b).\textsuperscript{99} Variants of this process exist.\textsuperscript{99}

Microstructured surfaces can also be produced in polymeric and similar materials by a replication process from a suitable cast. Average reflectances in the visible of the order of 0.3 percent have been reported for surfaces treated in this way (Fig. 19c).\textsuperscript{100}

Recently there has been a renewed interest in antireflection coatings in which the variable porosities of leached or etched layers are simulated by dense regular-shaped structures formed by photochemical or mechanical means. Clapham and coworkers appear to have been the first to demonstrate such devices.\textsuperscript{101} They applied a photoresist to a surface, exposed it to two orthogonal sets of ultraviolet interference fringes, and then

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure13.png}
\caption{Reflectances of three broadband antireflection coatings. (Curve 1, after Turner;\textsuperscript{408} curve 2, Optical Coating Laboratory;\textsuperscript{409} curve 3, Thin Film Lab.\textsuperscript{410}) Curve 4 (after Balzers\textsuperscript{406}) corresponds to a single-layer AR coating on glass and is shown here for comparison purposes.}
\end{figure}
OPTICAL PROPERTIES OF FILMS AND COATINGS

**FIGURE 15** Transmittance of plates of infrared materials antireflection-coated on both sides:
(a) Silicon—Curve 1: uncoated plate of 1.5-mm thickness; curve 2: single $\lambda_n/4$ layer of SiO$_2$ ($\lambda_n = 1.8 \mu$m); curve 3: $\lambda_n/4$ coatings of MgF$_2$, and CeO$_2$ ($\lambda_n = 2.2 \mu$m); curve 4: hard carbon layer; (b) Germanium. Curve 5: uncoated plate; curve 6: single $\lambda_n/4$ layer of SiO ($\lambda_n = 2.7 \mu$m); curve 7: $\lambda_n/4$ coatings of MgF$_2$, CeO$_2$, and Si ($\lambda_n = 3.5 \mu$m); curve 8: environmentally stable, high laser damage threshold three-layer design based on ThF$_4$ and Ge. (c) Irtran II. Curve 9: uncoated plate of 2-mm thickness; curve 10: single $\lambda_n/4$ coating of CeF$_3$; curve 11: $\lambda_n/4$ layers of MgF$_2$ and SiO ($\lambda_n = 4.2 \mu$m). (d) Zinc selenide. Curve 12: uncoated plate; curve 13: extremely broadband AR coating composed of 398 layers produced by molecular beam deposition. (*Curves 1–3, 5–7, 9–11 after Cox and Hass*; curve 4 after Balzers; *curve 8 after Oh*; *curve 13 after Fisher.*

**FIGURE 16** Calculated reflectances of the interfaces between two media antireflection-coated with inhomogeneous layers of indicated thickness and (complex) refractive indexes that vary smoothly, from the index of one medium to that of the other: (a) two nonabsorbing media of indexes 1.52 and 2.36; (b) glass-chromium interface with refractive indexes 1.52 and 2.26-0.43, respectively. (*After Anders and Eichinger.*)
developed it to form a regular array of protuberances that could be optionally enlarged by additional ion-beam etching. Such surfaces can reduce the reflectance to less than 0.3 percent in the visible part of the spectrum (Fig. 19f). The theory of such structures has been investigated by several workers and devices for wavelengths extending into the mm and sub-mm region have been fabricated.

Antireflection Coating of Absorbing and Amplifying Media

Antireflection coatings for glasses and semiconductors in regions of weak absorption, and for present-day laser materials in which \( k \), the imaginary part of the complex refractive index [Eq. (25)], is small and negative, differ little from those described previously.

The reduction of the reflectance of opaque materials for architectural, decorative, and
FIGURE 19 Measured performance of various broadband inhomogeneous antireflection coatings: (a) subtractive process—leached and etched phase-separation glass (after Asahara); (b) additive/subtractive process—leached and etched phase-separable film deposited onto fused silica (after Yoldas); (c) replication process on a cellulose acetate butyrate surface (after Maffitt); (d) structured antireflection coating in photosensit (after Wilson).

technical purposes leads to a corresponding increase in the absorption. This can be utilized to improve the efficiency of radiation detectors and to control the solar-absorption and thermal-emittance characteristics of surfaces (see, for example, Refs. 109 and 110). A measured example of the reduction in the reflectance of a metal surface attainable with a homogeneous nonabsorbing layer is shown in Fig. 20.

Inhomogeneous transition layers whose refractive index and extinction coefficient change gradually from the values of one of the media to those of the other are also very effective. The calculated reflectance of a glass-chromium interface coated in this way, useful for blackening of prism faces, lens edges, scales, etc., is shown in Fig. 16b. Metal-air interfaces can be treated a little less effectively with single layers because of the lack of suitable low-index materials.

FIGURE 20 Antireflection coating of opaque metals. Curves 1 and 2: reflectance of chromium and of chromium with a ZnS layer. (After Lupashko and Shkayevskii.)
Antireflection Coating of Surfaces Carrying a Thin Film

For some applications it is necessary to deposit a certain film onto a glass surface. The objectionable reflectance that such a film would normally introduce can be avoided by incorporating it into an antireflection coating. Thus, for example, Fig. 21 shows the calculated transmittances of two conducting layers before and after inclusion in antireflection coatings. The use of homogeneous and of inhomogeneous antireflection coatings with absorbing layers for ophthalmic purposes is described by Katsube et al. and Anders.

Antireflection Coatings at Nonnormal Angle of Incidence

The calculated performances at 0°, 45°, and 60° incidence of the three commercially most important antireflection coatings and of a 10-layer coating for germanium are shown in Fig. 22. The deterioration with angle of incidence is particularly severe for a narrowband
high-efficiency antireflection coating. Figure 22d suggests that the closer the design of an antireflection coating approximates an inhomogeneous transition layer (see "Inhomogeneous and Structured Antireflection Coatings" in Sec. 42.6), the less angle-dependent is its performance.

To design an antireflection coating for one angle of incidence and one plane of polarization, the effective thicknesses and refractive indices [Eqs. (18) and (19)] of its layers should satisfy the relations set out in Table 1. In practice, small departures from those conditions are required to optimize the performance with good coating materials. Calculated curves for two sets of two- and three-layer coatings designed for use at 45° are shown in Fig. 23.

If the obliquely incident radiation is unpolarized, a compromise is necessary. The effective thicknesses are matched for the design angle, but the refractive-index conditions set out in Table 1 are satisfied for normal incidence since they cannot be satisfied for both polarizations at the same time (Fig. 24).

Achromatic antireflection coatings can be designed that are optimized for both

---

**FIGURE 23** Calculated reflectance at 0° (solid curve), 45° (dashed curve), and 60° (dotted curve) of (a) and (c) two-layer, and (b) and (d) three-layer antireflection coatings designed for light incident at 45° and polarized parallel or perpendicular to the plane of incidence. (After Turbari. 418, 419)

**FIGURE 24** Calculated reflectance at 0° (solid curve), 45° (dashed curve), and 60° (dotted curve) of (a) two-layer and (b) three-layer antireflection coatings (systems 2.1 and 3.4 in Table 1) for use with unpolarized light, with the thicknesses of the layers optimized for an angle of incidence of 45°.
polarizations at the same time or that are suitable for use over a wide range of angles of incidence (Fig. 25b). But the problem becomes more difficult the greater the spectral and angular ranges required, especially if angles of incidence greater than 60° are involved. The performance of a single-wavelength antireflection coating that is effective for angles of incidence of up to 60° is shown in Fig. 26. The design of antireflection coatings for very high angles of incidence has been considered by Monga.

42.7 TWO-MATERIAL PERIODIC MULTILAYERS—THEORY

Nonabsorbing \([AB]^N\) and \([AB]_N\) Multilayer Types

Let a periodic multilayer be composed of \(N\) periods \(AB\), where \(A, B\) represent layers of refractive indices \(n_A, n_B\) and optical thicknesses \(n_A d_A, n_B d_B\). The most general representation of the complete multilayer is

\[
AB \cdot AB \cdot AB \cdots AB = [AB]^N
\]

In practice it is customary to write \([HL]^N\) or \([LH]^N\), depending on whether \(n_A\) is greater or less than \(n_B\), respectively. The spectral-reflectance curve of the multilayer \([AB]^N\) will lie

![Graph](image-url)
FIGURE 27 (a) Calculated reflectance and (b) phase change on reflection of periodic multilayers of the type [HL]_N. H and L stand for high- and low-refractive-index layers of quarter-wave thickness at \( \lambda_0 = 1.0 \) \( \mu \)m. The heavy lines are the envelopes of the reflectance curves.

within a pair of envelopes which, at normal incidence, depend only on \( n_A d_A, n_B d_B, n_A, n_B, n_s, \) and \( n_m, n_1, n_2, \ldots N \). For \( n_s = n_m \) the lower of the envelopes becomes \( R = 0 \). The envelopes contain high-reflectance zones within which the reflectance at each wavelength increases monotonically with the number of periods, approaching 1.0 as \( N \) tends to infinity (Fig. 27). Outside these high-reflectance zones the curves exhibit subsidiary maxima and minima whose number depends on \( n_A d_A/n_B d_B \) and which increases with \( N \). The first-order high-reflectance zone occurs at a wavelength \( \lambda_1 \) given by

\[
n_A d_A + n_B d_B = \frac{\lambda_1}{2}
\]

and subsequent zones occur at wavelengths \( \lambda_q (\lambda_1 > \lambda_2 > \lambda_3 \ldots) \) given by

\[
N(n_A d_A + n_B d_B) = q \frac{\lambda_q}{2} \quad q = 2, 3, 4, \ldots
\]

providing that at these wavelengths

\[
n_A d_A, n_B d_B \neq p \frac{\lambda_a}{2} \quad p = 1, 2, 3, \ldots
\]

It is thus possible, by choosing suitable-thickness ratios, to arrange for or to suppress high-reflectance zones in several spectral regions at the same time. This is useful in the design of broadband reflectors, cut-off filters, hot and cold mirrors (Sec. 42.9) and laser
reflectors, etc. Typical curves for thickness ratios 1:1 and 2:1 are given in Fig. 28. Plotted on a $\lambda_{o}/\lambda$ scale, they show symmetry about all wavelengths $\lambda$ for which $n_{A}d_{A}$ and $n_{B}d_{B}$ are both equal to some integral multiple of $\lambda/4$.

**Maximum Reflectance.** For a given refractive index ratio ($n_{A}/n_{B}$) and number of periods $N$, the highest reflectance occurs whenever $n_{A}d_{A}$, $n_{B}d_{B}$ are each equal to an odd multiple of $\lambda/4$. It is given by

$$R_{\text{max}} = \frac{\left[\frac{n_{a}}{n_{s}} - \frac{1}{n_{a} + \left(\frac{n_{A}}{n_{B}}\right)^{2N}}\right]}{\left[\frac{n_{a}}{n_{s}} + \left(\frac{n_{A}}{n_{B}}\right)^{2N}\right]}$$

Intermediate reflectances are obtained for the related symmetrical multilayers $[AB]^{N}A$:

$$R_{\text{max}} = \frac{n_{a}n_{s}n_{A}^{2} - \left(\frac{n_{A}}{n_{B}}\right)^{2N}}{n_{a}n_{s}n_{A}^{2} + \left(\frac{n_{A}}{n_{B}}\right)^{2N}}$$

Results for a number of such cases are given in Fig. 29. Intermediate reflectances can be obtained by changing the refractive index of any of the layers in the stack.

Explicit expressions for $R$ for other thicknesses are complicated.

**Phase Change on Reflection.** The dispersion of the phase change on reflection from periodic all-dielectric multilayers is much greater than that of metal reflectors. Unless corrected for, it will lead to errors in some metrological and interferometric applications. Like reflectance, it varies rapidly outside the high-reflection zone (Fig. 27b). Within the high-reflection zone, it changes almost linearly with wavenumber and is $180^\circ$ at $\lambda_{o}/\lambda = 1$. 

![Calculated reflectance curves of periodic multilayers with (a) 1:1 and (b) 2:1 thickness ratios. The dotted curves represent the reflectance of polarized radiation incident at an angle of 60°.](image-url)
FIGURE 29 Transmittance at the reflection maximum of quarter-wave stacks composed of MgF$_2$ and SiO$_2$ in the visible region (●), MgF$_2$ and MgO in the ultraviolet (■), MgF$_2$ and ZnS in the visible region (▲), and ZnS and PbTe in the infrared (○).

The slope of this portion of the graph increases and approaches a limiting value as $N$ tends to infinity. The limiting values (in degrees per unit wavelength) are given by

$$\frac{d\varepsilon}{d\lambda} \approx \frac{180n_m}{\lambda (n_H - n_L)} \quad \text{for } n_H$$

$$\frac{180n_m}{\lambda n_m(n_H - n_L)} \quad \text{for } n_L$$

depending on whether the light is first incident on a high-($n_H$) or low-($n_L$) refractive-index layer.$^{119}$ These values should be multiplied by 3, 5, ... in stacks composed of 3\(\lambda/4\), 5\(\lambda/4\), ... layers.

Böhme has shown that, by changing the refractive index of one of the layers in a quarter-wave stack, it is possible to obtain a zero value of the phase change on reflection $\varepsilon$ at $\lambda_o$. $^{120}$

Periodic Multilayers of the $[(0.5A)B(0.5A)]^N$ Type

The construction of such multilayers differs from that of the type $[AB]^N A$ discussed previously by having outermost layers of only half the thickness of the remaining layers.$^{69}$ The position and width of the high-reflection zones are the same in both cases, but in coatings of the $[(0.5A)B(0.5A)]^N$ type, it is possible to reduce the height of the subsidiary maxima on either one or the other side of the main reflectance maximum if $n_A d_A = n_B d_B$. Kard et al. describe the optimum choice of all the construction parameters ($N$, $n_A$, $n_B$, $n_s$, ...).
Optical Properties of Films and Coatings

Figure 30 Calculated reflectance curves of two periodic multilayers with symmetrical periods of the type \([0.5L]H[0.5L]^6\) in which the subsidiary reflectance maxima on the short- and the long-wavelength side of the high-reflectance zone are reduced.

If the refractive indices \(n_s\) and \(n_m\) of the surrounding media must be chosen on some other basis, then for maximum improvement of the short- and long-wavelength transmission, the refractive indexes \(n_A\) and \(n_B\) of the coating materials must satisfy

\[
n_A n_m = \frac{n_A^2}{n_B^2}
\]

or

\[
n_A n_m = n_A n_B
\]
respectively. Estimates of the maximum reflectance for larger values of \(N\) can be obtained from Fig. 29. Multilayers of this type are of importance in the design of cut-off filters (Sec. 42.9), and are illustrated in Fig. 30, which should be compared with the curves of Fig. 31.

Width of the High-Reflectance Zone

For a given value of \((n_A/n_B)\), the width \(\Delta \lambda_R/\lambda\) of the high-reflectance zone is greatest when \(n_A d_A = n_B d_B = \lambda/4\), and it is then given by

\[
\frac{\Delta \lambda_R}{\lambda} = \frac{4}{\pi} \sin^{-1}\left(\frac{1 - n_A/n_B}{1 + n_A/n_B}\right)
\]

This width is reduced by a factor of \(2N - 1\) if \(N\)th-order quarter-wavelength layers are used. Graphs of the widths of the high-reflectance zones vs. refractive-index ratio for \(\lambda/4\), \(3\lambda/4\), and \(5\lambda/4\) layer stacks are given in Fig. 32. Periodic multilayers with equal refractive-index ratios have high-reflectance zones of equal widths, but their reflectance curves will not be the same unless the refractive indices of the surrounding media are also increased by the same ratio (Fig. 31).

Periodic Multilayers of the \([xH \cdot (1 - x)L]^N \cdot xH\) Type

As already stated previously, it is not necessary for the optical thicknesses of the layers of a periodic multilayer reflector to be equal to a quarter-wave. A multilayer system of the type \([xH \cdot (1 - x)L]^N \cdot xH\), where \(H, L\) correspond to quarter-wave layers of a high and low refractive index and where \(0 < x < 1.0\), will have a high reflectance, providing that the
FIGURE 31 Calculated reflectance curves of two quarter-wave stacks of the type $H(LH)^N$ with different values of $n_H$ and $n_L$, but with the same ratio $n_H/n_L$, deposited onto two different substrate materials. The dotted curves represent the reflectances for light incident at 60°.

FIGURE 32 Calculated widths of high-reflectance zones of two-material periodic stacks of layers of optical thicknesses $\lambda/4$, $3\lambda/4$, and $5\lambda/4$ for different refractive-index ratios.
OPTICAL PROPERTIES OF FILMS AND COATINGS

FIGURE 33 Maximum reflectance (a) and width of the high-reflectance zone (b) of periodic multilayers of the type \([xH \cdot (1-x)L]^N xH\). The curves were evaluated for refractive indices \(n_H = 2.4\) and \(n_L = 1.51\). (After Shao.)

Angular Sensitivity

For some applications it is important to reduce the angular variation of the reflectance curve. This can be done by using materials with high refractive indices (Fig. 31) (see also “Angle-of-incidence Effects” in Sec. 42.9). Another way is to use periods in which the high-index film is thicker than the low-index film (Figs. 28b and 57a).

Multilayer Reflectors Made of Absorbing Materials

It is possible to achieve a high reflectance with multilayers of the \([AB]^N A\) type even when \(A, B\) correspond to absorbing materials. This is of particular interest for spectral regions for which no nonabsorbing coating materials exist. The optical thicknesses of the periods of such systems are still approximately equal to \(\lambda/2\) but, for maximum reflectance, the individual thicknesses \(d_A, d_B\) may be quite different, depending on the number of periods \(N\) and on the optical constants of the materials used. Reflectors with \(k_A > k_B\), have structures that are intermediate to those of quarter-wave stacks (with \(k_A = k_B = 0\) and optical thicknesses of \(\lambda/4\), in which constructive interference effects are maximized), and those of ideal Bragg crystals (with \(k_A \gg k_B\), and in which \(d_A \approx d_B\) to minimize absorption losses).

In the extreme ultraviolet (XUV) and in the soft x-ray regions, \((n - 1)\) and \(k\) are much smaller than 1 for most coating materials. To design a periodic multilayer with a high-normal-incidence reflectance for a given wavelength, it is first necessary to choose a material \((n_H - ik_H)\) with the lowest possible extinction coefficient. Next, a second, chemically compatible material \((n_A - ik_A)\) is selected with the lowest extinction coefficient.
that will maximize the normal-incidence Fresnel reflection coefficient of the interface between the two materials given by:

$$r_{BA} = \frac{(n_B - n_A) - i(k_B - k_A)}{(n_B + n_A) + (k_B + k_A)}$$

Vinogradov and Zeldovich use a factor $\beta_{opt}$ to relate the metric thicknesses of the layers $A$ and $B$ that yield a maximum reflectance to the overall thickness $d_{opt}$ of the period:

$$d_A = \beta_{opt} \cdot d_{opt} \quad \text{and} \quad d_B = (1 - \beta_{opt}) \cdot d_{opt}$$

where $\beta_{opt}$ is obtained by the solution of the equation

$$\tan(\pi \beta_{opt}) = \pi \left[ \beta_{opt} + \frac{n_B k_B}{n_A k_A - n_B k_B} \right]$$

$d_{opt}$ is approximately equal to $\lambda/2$, but Vinogradov and Zeldovich give a more accurate expression for this quantity, as well as for the limiting reflectance $R$ and the number of periods $N$ required to reach that value.

### 42.8 MULTILAYER REFLECTORS—EXPERIMENTAL RESULTS

In the calculations of the data for Figs. 27 to 33, the dispersion of the optical constants of the materials was ignored, and it was assumed that the films were absorption- and scatter-free and that their thicknesses had precisely the required values. In practice, none of these assumptions is strictly valid, and hence there are departures from the calculated values. In general, the agreement is better within the high-reflectance zone than outside.

**Reflectors for Interferometers, Lasers, Etc.**

The measured reflectances and transmittances of a number of quarter-wave reflectors suitable for use in Fabry-Perot interferometers are shown in Fig. 34. The transmission of a

![FIGURE 34 Measured spectral reflectance and transmittance curves of periodic all-dielectric reflectors of the type $[HL]^N H$ for the ultraviolet, visible, and near-infrared parts of the spectrum. Curve 1: 27 layers of MgO and MgF$_2$; curves 2 and 5: 11 and 13 layers of PbF$_2$ and cryolite, respectively; curves 4, 5, 7, and 8: 7 layers of ZnS and cryolite; curves 6, 9, and 10: 9 layers of ZnS and cryolite. (Curve 1 after Apfel,[423] curves 2 to 10 after Heffet et al. [424]).](image-url)
typical commercial laser reflector for $\lambda = 0.6328 \mu m$ is shown in Fig. 35. The measured reflectances of a number of highly reflecting coatings for the infrared spectral region are shown in Fig. 36.

Both “soft” laser coatings that can be dissolved in weak acids and “hard” coatings that can be removed only through polishing are available commercially.

Effects of Imperfections

**Thickness Errors.** Small errors in the thickness of the layers of a quarter-wave stack have only a very small effect on the reflectance and on the phase change on reflection of the multilayer within the high-reflectance zone, but they do affect the performance outside the zone.$^{130,134}$ In fact, the effect may be quite serious: thickness variations can give rise to an apparent lack of flatness of the substrate surface.$^{132–134}$

**Dispersion.** The most noticeable effects of dispersion in quarter-wave stacks of a given multilayer type are the increase in the peak reflectance with decrease in wavelength (see Fig. 34) and the asymmetry of the maxima on either side of the main reflectance maximum.
Absorption. The separation between the transmission and reflectance curves in Fig. 34 is mostly due to absorption. These losses can limit the usefulness of the reflectors for some applications. Thus, for example, in interference filters and Fabry-Perot interferometers they lead to a reduction in the peak transmissions and limit the attainable half-widths. In optical information-storage devices they set a limit to the highest reflectance attainable. In lasers, the losses counteract directly the gain in the laser medium. In addition, absorption within the layers is responsible for damage to laser reflectors (see also “Laser Damage” in Sec. 42.5).

If the two materials used for the construction of periodic quarter-wave stacks have small but finite extinction coefficients, the resulting absorption at first reduces both the transmission and reflection coefficients.\(^{117,132,135,136}\) With an increase in the reflectance of a multilayer of the \([HL]^N H\) type, the absorption occurs more and more at the expense of the reflection coefficient, approaching a limiting value of

$$A = -\delta R = \frac{2\pi m}{n_H - n_L} (k_H + k_L)$$

that is independent of the number of layers.\(^2\) The corresponding expression for a multilayer \([HL]^N\) in which a low refractive index faces the incident medium is

$$A = -\delta R = \frac{2\pi (n_H^2 k_H + n_L^2 k_L)}{n_m (n_H^2 - n_L^2)}$$

Figure 37 shows the spectral variation of experimentally determined maximum reflectances and limiting losses of various quarter-wave multilayer reflectors.

The absorption in periodic multilayers composed of materials having finite extinction coefficients can be reduced below the values given by Eqs (52) and (53) if a structure of the \([xH \cdot (1-x)L]^N \cdot xH\) type is used.\(^{124}\)

![Figure 37](image-url)
Surface and Interface Imperfections. In thin-film calculations, it is usually assumed that substrate surfaces and interfaces between the layers are smooth and that the layers are homogeneous. In practice, substrate surfaces and interfaces have a certain roughness and, at times, thin uniform or inhomogeneous interface layers formed between the boundaries of two layers as a result of oxidation, chemical interactions, or interdiffusion of the two coating materials. The interface layers, as a rule, are different at the AB and BA boundaries and are typically 0.0003 or 0.0010 μm thick. These and other imperfections of the layer system result in reduced reflection and/or in scatter. If ignored in the model used to represent the multilayer, they add to the discrepancies that are observed between the calculated and experimental data.

Scattering losses are particularly significant at shorter wavelengths. For this reason, there have been many theoretical and experimental investigations of scattering of surfaces and thin films. Investigations have shown that when scatter does occur, most of the light is scattered in directions that are close to that of the specularly reflected light. The experimental results for a typical mirror are shown in Fig. 38.

Very Low Loss Reflectors. Mirrors with very low losses are required for use in laser cavities and in ring lasers. Mirrors with a combined loss $L = \text{transmission + absorption + scatter}$ of the order of $5 \times 10^{-6}$ are commercially available. With special manufacturing techniques, practically loss-free reflectors can now be made.

Recently a 41 quarter-wave stack made of Ta$_2$O$_5$ and SiO$_2$ layers with a combined loss of $L = 1.6 \cdot 10^{-6}$ corresponding to a reflectance of 0.9999984 at 0.633 μm, has been reported. The absorption and scatter losses were estimated to be of the order of $1.1 \cdot 10^{-6}$. The essential starting point for the manufacture of such coatings are superpolished substrates with a surface roughness of 0.5-Å rms or less. The layers were deposited by reactive ion-beam sputtering from high-purity oxide targets in a cryogenically pumped, fully automated deposition system.

Multilayers for the Soft X-ray and XUV Regions. The effect of roughness and of interface layers is particularly important in soft x-ray and XUV multilayers because the dimensions of these defects are comparable to the thicknesses of the individual layers. For this reason, much attention has been focused on the proper modeling of such coatings. Many workers use several very thin layers and the matrix method outlined in Sec. 42.3 to model roughness and interface layers. Others make use of the following recursive formula for the amplitude reflectance $r_i$ of the first $j$ layers of the system:

$$r_j = \frac{r_{j+1} + r_{BA} \exp (2i\delta_j)}{1 + r_{j+1}r_{BA} \exp (2i\delta_j)}$$  \hspace{1cm} (54)
Here $\delta_j$ is the effective optical thickness of the $j$th layer (Eq. 18), $r_{j-1}$ is the amplitude reflectance of the first $(j-1)$ layers, and $r_{BA}$ is the amplitude reflectance of the interface between the $j$th and $(j+1)$th layers. In this approach, if the Fresnel amplitude reflection coefficient $r_{BA}$ is replaced by

$$r_{BA} \exp \left( -\frac{1}{2} \frac{4\pi}{\lambda} \sigma \text{Re}(\bar{n} \cos \theta)\right)$$  \hspace{1cm} (55)$$

the combined effect of roughness and of interface layers $\sigma$ can be allowed for.\textsuperscript{141} In the hard x-ray region, where $n = 1$, $k = 0$ for all materials, the exponential term in the preceding expression reduces to the so-called Debye-Waller factor $DW$,

$$DW = \exp \left[ -\frac{1}{2} \left( \frac{4\pi}{\lambda} \sigma \cos \theta \right)^2 \right]$$  \hspace{1cm} (56)$$

The calculated reflectance of an XUV mirror, with and without the effect of surface imperfections, is shown in Fig. 39.

### Narrowband Reflection Coatings

Narrowband rejection filters transmit freely all the radiation incident upon them except in one narrow spectral region in which the radiation is either wholly or partially reflected.\textsuperscript{142} Lord Rayleigh observed a corresponding natural phenomenon in potassium chlorate crystals.\textsuperscript{143} Subsequent experimentors reported crystals with rejection bands varying between 0.001 and 0.038 microns in width and reflectances between 33 and 99.9 percent.\textsuperscript{144–146} But, unfortunately, at present the size of the crystals that can be grown is insufficient, and the position and width of the rejection region cannot be controlled. The same comments can be made about coextruded polymer films made of the thermoplastic materials polypropylene ($n = 1.49$) and polycarbonate ($n = 1.59$).\textsuperscript{147,148} With thin films, these limitations can be overcome, but it is difficult to achieve the extremely narrow widths and high rejections observed in the crystals.

A quarter-wave stack of the type $[AB]^N A$ has been suggested as a model for the construction of such filters (see, for example, Refs. 145 and 149). It follows from Figs. 29 and 32 that the closer the refractive index ratio $n_A/n_B$ is to unity, the narrower the width of the reflectance zone and the more layers required to achieve a certain rejection. Shown in
Optical Properties of Films and Coatings

**Fig. 40** Measured reflectance of a 720-layer quarter-wave stack produced by chemical vapor deposition using materials with refractive indices of 1.575, 1.585. (After Edmonds et al.)

Fig. 40 is the measured reflectance of a multilayer consisting of 720 layers that was produced by a plasma chemical deposition technique. Unfortunately, the method could only deposit such coatings on the inside of a tube. To reduce the number of layers, films with higher $n_A/n_B$ ratios could be utilized and the width reduced by using layers with thicknesses that are odd multiples of a quarter-wavelength. But this can be done only at the expense of bringing the adjacent higher- and lower-order reflection peaks closer. Resonant reflectors are an extreme example of this.

**Resonant Reflectors**

Even “hard” evaporated coatings cannot survive the power densities present in some high-power lasers. In the past, resonant reflectors have been used. They consist of one or more accurately air-spaced plane-parallel plates of thicknesses of the order of millimeters made of a tough, high-optical-quality material. Because of the long coherence length of the laser radiation incident upon them, interference takes place in the same way as in thin films. Resonant reflectors may be regarded as being quarter-wave reflectors of enormously high order of interference, and all the equations given in Sec. 42.7 apply.

Resonant reflectors made of quartz and sapphire are commercially available. Since the refractive index of quartz is lower than that of sapphire, a larger number of plates is required to attain the same reflectance. On the other hand, quartz is much cheaper and is less temperature-sensitive. The calculated reflectance of one-, two-, three-, and four-plate sapphire resonant reflectors are shown in Fig. 41. In another development, diffusion-doped quartz plates are used in the construction of resonant reflectors. The doping process causes the refractive index of the plate to increase smoothly in a 0.5-μm region from that of quartz to about 2.0 at the surface. As a consequence, fewer elements are needed to achieve a given reflectance.

**All-dielectric Broadband Reflectors**

There are several ways of obtaining a coating with a broad high-reflection region should the width attainable with quarter-wave stacks be inadequate. For a broad region with a very high-reflectance, one can superimpose two quarter-wave stacks tuned to two different wavelengths. The widest continuous high-reflectance region is attained when materials with the highest available refractive-index ratios are used and the thicknesses of the layers are so chosen that the two high-reflectance zones are contiguous (see Fig. 42c). For an even broader region, further quarter-wave stacks can be superimposed. If high-reflectance regions overlap, special precautions must be taken to prevent the appearance of sharp reflectance
MINIMAS IN THE HIGH-REFLECTANCE REGION. It is not easy to obtain a very uniform, moderately high reflectance in this way. Another approach is to deposit onto the substrate a series of alternating high- and low-refractive-index films of gradually increasing or decreasing thicknesses (Fig. 43). A broad high-reflection region can also be obtained with a multilayer in which the layers are different multiples of λ/4 of a selected wavelength (see Fig. 44). Finally, it has been suggested that multilayers with 10:1 high-reflectance regions might be produced by depositing hundreds of layers of random thicknesses made of two materials that are nonabsorbing throughout the spectral range of interest. As yet, experimental data for this approach have not been presented.

If only a relatively small increase in the high-reflectance region is required, or if a uniform but only moderately high reflectance is required, the desired result can be achieved by modifying the thicknesses and refractive indices of a quarter-wave stack using a computer refinement program or by the addition of achromatizing λ/2 layers. The measured performances of several such reflectors are shown in Figs. 45 and 46. Broadband
FIGURE 43 Calculated reflectance of an all-dielectric broadband reflector consisting of 35 layers made of a low- and a high-index material and having optical thicknesses that vary in a geometric progression. (After Heavens and Liddell.\textsuperscript{427})

FIGURE 44 Calculated reflectance of an all-dielectric broadband reflector consisting of 11 layers all of which have optical thicknesses that are various multiples of 0.13 μm. (After Elsner.\textsuperscript{428})

FIGURE 45 Measured reflectances of all-dielectric broadband reflectors designed with refinement programs. [(a) after Penselin and Steudel,\textsuperscript{429} (b) after Baumeister and Stone,\textsuperscript{430} (c) after Ciddor,\textsuperscript{431} (d) after Ramsey and Ciddor.\textsuperscript{134}]
reflectors with moderate and high reflectances for the ultraviolet spectral region have been reported by Korolev, Sokolova, and Stolov.

Phase Change on Reflection from Broadband Reflectors. The phase change on reflection from broadband reflectors varies even more rapidly with wavelength (Figs. 42 to 46) than that from quarter-wave stacks (Fig. 27). This can be a disadvantage in certain metrological applications. Another consequence of this rapid variation is that, in the presence of systematic thickness variations of the layers, it can give an impression of lack of flatness in the substrate. Figures 45 and 46 represent designs of broadband reflectors in which an effort was made to reduce this effect.

Rejection Filters

Minus Filters. Minus filters are, in essence, multilayer reflectors in which the ripples in the transmission regions on either side of the high-reflectance zone have been reduced or eliminated. Filters of this kind with various widths and attenuations find applications as correction filters. Narrow minus filters with high attenuations have various scientific and technological uses, including protection of equipment and personnel from harmful laser radiation. Figure 47 shows the measured transmittances of a number of rejection filters of various widths and attenuations.

Thelen has shown how to optimize the transmission on both sides of the rejection band of a minus filter simultaneously. If the multilayer is surrounded on both sides by the same medium, it will be symmetrical and can be represented by $C[AB]^{AC}$.
FIGURE 48 Suppression of ripples in the transmission region of rejection filters: (a) calculated spectral transmittance curves and refractive index profiles of a 9-layer quarter-wave stack; (b) a 17-layer, 5-material minus filter; and (c) a two-material equivalent of the minus filter.

DAC/[AB]^[nACAD]... Here A, B, C, D,... are layers of quarter-wave optical thickness at the design wavelength and

\[ n_m = n_s = n_A \]

The refractive indices \( n_c, n_d, \ldots \) depend on \( n_A \) and \( n_B \) in a more complicated way. The larger the number of different materials used in the construction of these multilayers, the better the performance in the transmission region. Should the use of more than two coating materials not be convenient, it is a simple matter to find a two-material version of this solution. These points are illustrated in Fig. 48.

Young\textsuperscript{142} described two other design methods for narrowband rejection filters with improved transmission characteristics. The methods are based on analogies with antenna theory, and they yield nonperiodic equiripple designs in which all the layers either have equal thicknesses but different indices or are made of two materials only but have many different thicknesses.

Should the simultaneous rejection of several wavelengths be required, it is possible to achieve this by depositing several minus filters on top of one another (Fig. 49).

**Rugate Filters.** In inhomogeneous layers, the refractive index varies continuously in the direction of the thickness of the layer.\textsuperscript{17,166,167} If the refractive index varies in a periodic manner between two extreme values, it is possible to design a minus filter with a high transmission on either side of the rejection band (Fig. 50). Such periodic inhomogeneous layers are sometimes called rugate filters. Some workers reserve this term for inhomogeneous layers in which the logarithm of the refractive index varies in a sinusoidal manner. The rejection wavelength corresponds to that wavelength for which the period of
the index variation is equal to a half-wavelength. The attenuation depends on the ratio of the highest to lowest refractive index in the design and on the number of periods. As in the multilayer minus filters, the width of the rejection region also depends on the refractive index ratio. Rugate filters do not have the higher-order reflection peaks that are characteristic of periodic multilayers and this is one reason for their attractiveness. However, they are more difficult to produce. If necessary, they can be approximated by a homogeneous multilayer system consisting of a few materials.

As in the case of minus filters, it is possible to reject a number of wavelengths by depositing several rugate filters on top of each other. However, the combined overall
Simultaneous suppression of several laser wavelengths: (a) calculated spectral transmittance curves and refractive index profiles of a series and (b) a parallel solution to the problem. (After Verly.)

Thickness of the rugate filters will then be quite high. It is possible to find an inhomogeneous layer solution to this problem in which the refractive index profile is more complicated, but which requires a considerably thinner inhomogeneous layer (Fig. 51). In Liipmann-Bragg holographic mirrors, the refractive index varies continuously in a direction perpendicular to the plane of the substrate. These devices behave like thin-film systems and have properties similar to those of rugate filters. Holographic edge and narrowband rejection filters are available commercially.

Graded Reflectivity Mirrors

Mirrors in which the absorption or reflection varies radially have been proposed in the past for the control of modes and of edge diffraction effects in laser resonators. In addition to meeting the reflectance specifications, graded reflectivity mirrors must have a sufficiently high laser damage threshold for use with high-power lasers. Graded reflectivity mirrors are produced by depositing thin films through a suitable mask. The substrate and the mask can be stationary, or one or both can rotate. A single shaped layer suffices for a maximum reflectance of intermediate values. For higher values, the shaped layer can be inserted between a stack of layers of uniform thickness or, alternatively, all the layers can be deposited through the mask. The experimentally measured reflections of two graded reflectivity mirrors are shown in Fig. 52.

Multilayer Reflectors for the Far-infrared Region

Thin-film filters cannot be produced by conventional deposition techniques for wavelengths greater than about 80 μm because of a lack of low-absorption coating materials that can be deposited in the form of thick, stable films. However, a hybrid process in which plastic sheets coated with relatively thin high-refractive-index materials are heat-bonded can be used to produce self-supporting optical multilayer filters. As mentioned before, periodic multilayers of unequal optical thickness can have a high reflectance, providing that the number of periods is high enough. The measured reflectance curves of two typical hybrid multilayer filters are shown in Fig. 53.
Multilayer Reflectors for the Soft X-ray and XUV Regions

There are two main obstacles to obtaining multilayers with a high-normal-incidence reflectance in the soft x-ray and in the XUV regions. First, at these wavelengths all materials absorb; this limits the number of layers that can contribute to the overall reflectance. Second, roughness and the interdiffusion and alloying of the materials degrade the individual interfaces; this reduces their contribution to the overall reflectance.

XUV mirrors are normally produced from two chemically compatible materials by sputtering or by electron beam gun evaporation. As already discussed, the XUV multilayer mirrors have a period of optical thickness $\lambda/2$. Within this period, the thickness of the less-absorbing material is larger. This material usually has an absorption edge that is close to the design wavelength. Optical constants can vary widely in this region. The second, more absorbing material is, therefore, selected to maximize the Fresnel reflection coefficient of the interface.

Theoretically, the best results are obtained with pure elements. However, sometimes alloys are used because they result in multilayers with better interfaces. Thus, MgSi$_2$ might be used in place of Mg, or B$_4$C instead of C or B. Examples of more commonly used material pairs are: (1) Mo/Si for the 130- to 250-Å region; (2) Mo/Y for the 90- to 130-Å region; (3) W/B$_4$C, Ru/B$_4$C, Mo/B$_4$C, etc. for the 70- to 130-Å region; and (4) Co/C, W/C, Re/C, ReW/C, Ni/C, etc., for the 45- to 70-Å. In these pairs, the second material in each pair has the lower extinction coefficient.

There are many reasons why there are differences between the theoretical performance of a multilayer reflector and the reflectance measured on a synchrotron. The highest...
FIGURE 54  Measured spectral reflectance curves of some representative multilayer x-ray mirrors. The materials used, the number of layers $N$ and the angles $\theta$, at which the reflectance was measured are indicated in the table. (Curves 1 after Spiller,\textsuperscript{435} curves 2–4 after Montcalm et al.,\textsuperscript{436,437} curves 5 and 6 after Zwicker,\textsuperscript{438} curve 7 after Ceglio,\textsuperscript{439} curve 8 after Falco.\textsuperscript{440})

reflectance achieved thus far is $R = 0.65$ at 135 Å for a Mo/Si multilayer.\textsuperscript{141} The measured reflectance of other experimentally produced x-ray and XUV mirrors is shown in Fig. 54.

42.9 CUT-OFF, HEAT-CONTROL, AND SOLAR-CELL COVER FILTERS

Ideal cut-off filters would reject all the radiation below, and transmit all that above a certain wavelength, or vice versa. Real cut-off filters, of course, are not perfect and so, in addition to the cut-off wavelength, the slope of the transition region and the extent and average transmission values of the transmission and rejection regions must be specified. The tolerable departures of these quantities from the ideal values depend greatly on the application. Most all-dielectric cut-off filters are based on periodic multilayers, whose basic properties were described in Sec. 42.7).

Transmission in the Passband Region

The usual way of avoiding the secondary transmission minima in the transmission band of a quarter-wave stack is through the use of eighth-wave layers on both sides of the stack (see “Periodic Multilayers of the $(0.5A)B(0.5A)$ Type” in Sec. 42.7).

Other, less frequently used methods of smoothing the transmission in the passband are the adjustment of the thicknesses of all the layers of a quarter-wave stack,\textsuperscript{177} the use of homogeneous\textsuperscript{178} and inhomogeneous\textsuperscript{179} layers on either side of the stack; the choice of an optimum set of refractive-index values for the substrate and films,\textsuperscript{180} and the use of an equiripple design\textsuperscript{142} in which thicknesses are kept constant but the refractive indices are varied.

The Width of the Transmission Region

For short-wavelength cut-off filters of the type described here, the transmission region is limited only by the transmission characteristics of the materials used for their construction. In long-wavelength cut-off filters, the transmission regions can be limited by the appearance of higher-order reflectance maxima (see “Nonabsorbing $[AB]^n$ and $[AB]^nA$
Multilayer Types" in Sec. 42.7). Should this be a serious limitation, it is possible to suppress a number of adjacent reflectance maxima by using multilayers with periods composed of three or more different materials (Fig. 55). By using a period consisting of an inhomogeneous layer with a special refractive-index profile, an even larger number of consecutive reflectance maxima can be suppressed (see also Fig. 50). Figure 56 shows the measured results for two such experimental coatings in which reflectance maxima are

![Figure 55](image1.png)

**Figure 55** Calculated reflectance of three-material periodic multilayers with suppressed higher-order reflectance maxima. (After Thelen.441)

![Figure 56](image2.png)

**Figure 56** Measured transmittance of periodic multilayers in which higher-order reflectance maxima are suppressed through the use of periods that consist of an inhomogeneous layer. (After Scheuerman.188)
suppressed at three and nine consecutive integer multiples of $1/\lambda_{\text{c}}$, a fact obscured in the case of the second filter by the absorption of the materials used.

Transmission in the Rejection Region

Figure 29 can be used for an estimate of the number of layers required to achieve a given transmission (see “Maximum Reflectance” in Sec. 42.7). Through the use of suitable substrate materials, the transmittances throughout the rejection region can typically be below 0.01 and 0.1 percent for short- and long-wavelength cut-off filters, respectively. Rejection filters with higher rejections can be provided or, alternatively, two or more filters in series can be used if they are placed at a small angle to one another.

The Width of the Rejection Region

The width of the high-reflection region of $[(0.5\lambda A)B(0.5\lambda A)]^N$ coatings can be estimated from Fig. 32. There is no shortage of absorbing materials should it be necessary to extend the rejection region of short-wave cut-off filters. The number of suitable absorbing materials for long-wavelength cut-off filters is more restricted and often it is necessary to use thin films to extend the cut-off region. In addition to the superposition of two or more cut-off filters tuned to different wavelengths (see “All-Dielectric Broadband Reflectors” in Sec. 42.8), one can deposit such coatings on different substrates or onto the opposite sides of the same substrate. The resulting transmission will be governed by the considerations of “Transmission Filters in Series and Parallel” in Sec. 42.2.

Slope of the Cut-off

This quantity is defined in a number of ways. One common definition is

$$\frac{\lambda_{0.8} - \lambda_{0.05}}{\lambda_{0.5}} \times 100\%$$

(58)

where $\lambda_{0.8}$, $\lambda_{0.5}$, and $\lambda_{0.05}$ refer to the wavelengths at which the transmittances are 0.8, 0.5, and 0.05 of the maximum transmittance of the filter. Explicit formulas for the slope are complicated. The slope increases with the number of periods and with the refractive-index ratio. Slopes with values of the order of 5 percent are readily available in practice.

Angle-of-incidence Effects

The edges of cut-off filters move towards shorter wavelengths as the angle of incidence is increased. The use of higher-index materials reduces the effect. Measured results for a cut-off filter in which the shift was reduced by using high-refractive-index layers that have three times the thickness of the low-index layers are shown in Fig. 57a. See also “Nonpolarizing Edge and Bandpass Filters” in Sec. 42.10 on polarization-independent color-selective beam splitters.

Experimental Results

The spectral transmittance curves of a number of commercially available short- and long-wavelength cut-off filters are shown in Figs. 58 and 59. High performance long- and short-wavelength cut-off filters are depicted in Figs. 60 and 100. Similar filters for
**FIGURE 57** Effect of (a) incidence and (b) temperature on the performance of cut-off filters. (Optical Coating Laboratory)

**FIGURE 58** A series of commercial short-wavelength cut-off filters. (Curves 1, 5, 8, 9, 10, Optical Coating Laboratory; curve 2, Bausch & Lomb; curves 3 and 6, after Turner; curve 4, Eastman Kodak; curve 5; Infrared Industries)

**FIGURE 59** A series of commercial long-wavelength cut-off filters. (Curve 1, after Apfel; curve 2, Eastman Kodak; curves 3, 4, 7, Optical Coating Laboratory; curves 5 and 6, Infrared Industries)
intermediate wavelengths can, of course, be constructed. It is also possible to construct edge filters in which the thicknesses of all the layers vary in proportion around the circumference of the substrate (see also “Linear and Circular Wedge Filters” in Sec. 42.12). A tuning of the cut-off wavelength is thus possible.

**Heat Reflectors, Cold Mirrors, and Infrared-Suppressing Filters**

Only 39 percent of the total radiation from carbon arcs and 13 percent from tungsten lamps operated at 3250 K represent visible light. Most of the remaining energy is infrared radiation, which is converted into heat on absorption. The use of heat reflectors and cold mirrors in film projectors, in spot lamps for television and film studios, and in other optical instruments can lead to a very significant reduction of this unwanted heat.

Heat reflectors (also called hot mirrors) are special long-wavelength cut-off filters with a cut-off at 0.7 μm which transmit the visible radiation from 0.4 to 0.7 μm without disturbing the color balance. The width of the rejection region depends on the light source to be used and on whether a heat-absorbing glass is also to be used. The spectral-transmittance curves of three typical commercial heat reflectors are shown in Fig. 61a. The measured spectral-transmittance and reflectance curves of two heat-reflecting coatings not based on periodic multilayers are shown in Fig. 62.

Cold mirrors reflect as much as possible of the visible light incident upon them and transmit the remaining radiation. The reflectance curves of two commercial cold mirrors are shown in Fig. 61b.
Solar-cell Covers

Solar-cell covers remove the incident solar energy that does not contribute to the electrical output of the cell and protect it from possible deterioration of its performance through the action of ultraviolet radiation. The spectral-transmittance of a blue-red solar-cell cover is shown in Fig. 61b. The earlier blue solar-cell covers (curve 2, Fig. 58) protected the cell only from the adverse effects of ultraviolet radiation.

Temperature Effects

Refractive indices of optical materials increase almost linearly with increase in temperature, thus causing cut-off edges to move towards longer wavelengths. In actual filters, the fractional-wavelength shift varies between $3 \times 10^{-3}$ and $10^{-2}$/°C. Ion-plated films have a smaller temperature shift than films prepared by conventional e-beam evaporated layers. Higher-index materials tend to be more temperature sensitive, thus making it difficult to construct filters that are insensitive both to angle of incidence and temperature changes. The measured performance of a cut-off filter at two temperatures is shown in Fig. 57b.

Metal-dielectric Reflection Cut-off Filters

It is possible to construct metal dielectric cut-off filters that act in reflected light (Fig. 63). Short-wavelength cut-off filters consist of an opaque metal layer and one or more
additional layers. The light is removed through absorption within the absorbing layers of the system. The thicknesses of the individual layers are adjusted to maximize the absorption and width of the rejection region. Long-wavelength cut-off filters consist of all-dielectric multilayer reflectors superimposed onto a black absorbing coating. A high attenuation of the unwanted radiation can be achieved by placing identical filters of either type in a multiple-reflection arrangement of the kind depicted in Fig. 3.

Cut-off Filters Based on Absorption

All materials used in multilayer interference coatings possess short- and long-wavelength absorption edges. These can often be used to assist in the blocking of such filters. The admixture of small amounts of absorbing materials to an evaporant or organic coating solution is used at times to tune this absorption edge (Fig. 64a and c). For example, antireflection coatings containing such ultraviolet-absorbing materials can be used to protect works of art.

A series of commercial short-wavelength cut-off filters for the infrared spectral region consisting of chemically deposited silver sulfide coatings on silver chloride substrate substrates are also shown in Fig. 64b. These filters are quite delicate. When protected with a polystyrene layer, their transmittance is reduced and sharp absorption bands appear. The filters should not be used outdoors unless additionally protected, nor should they be exposed to ultraviolet radiation or temperatures in excess of 110°C.

**FIGURE 64** Spectral transmittance of absorbing films produced in various ways: (a) envelopes of the transmission maxima of thick evaporated films of ZnS (curve 2), Ge (curve 6), and various mixtures of ZnS and Ge (curves 3 to 5) on a glass substrate (curve 1). (After Chang452; (b) spectral transmittance of chemically deposited silver sulfide coating on silver chloride substrates (curves 7 to 11) (Eastman Kodak445); (c) intrinsic transmission of thin films of titanium dioxide with admixtures of heavy metal oxides, deposited from organic solutions: curve 12, TiO₂ + 1.5SiO₂; curve 13, TiO₂; curve 14, TiO₂ + 0.5PbO; curve 15, TiO₂ + 0.15Fe₂O₃; curve 16, TiO₂ + 5.7UO₃ (After Schröder453).
The greatest advantage of cut-off filters based on absorption in thin films is their much smaller angular dependence.

### 42.10 BEAM SPLITTERS AND NEUTRAL FILTERS

**Geometrical Considerations**

Several different physical forms of beam splitters are illustrated in Fig. 65. The simplest (Fig. 65b) consists of a coating on a transparent plane-parallel substrate. If the two derived beams are to traverse identical paths, a cemented beam splitter is used (Fig. 65c). The
lateral displacement of the transmitted beam introduced by these forms can be avoided with a beam-splitting cube (Fig. 65f). To reduce the stray reflected light in the system, the free surfaces of these beam splitters can be antireflection-coated. Alternatively, the coatings can be deposited onto an approximately 2-μm-thick nitrocellulose pellicle (Fig. 65a). The latter is an integral part of the multilayer and may introduce an interference pattern into the spectral reflectance and transmittance characteristics. Pellicle beam splitters are very light and yet quite sturdy. They are, however, subject to vibrations caused by air currents and acoustical waves. The mechanical design of rugged, environmentally stable mounts for these types of beam splitters have been discussed by Heinrich et al., and Lipshutz. Pellicles made of Mylar have been used at temperatures down to 4 K.

In general, the transmission and reflection coefficients $T$ and $R$ will depend on the polarization of the incident radiation. The polarization effect can be reduced through the use of more complicated thin-film designs, but usually at the expense of other performance aspects. Achromatic or color-selective beam-splitting arrangements have been described in which the two derived beams have intensities that are completely polarization-independent over a very wide spectral region. They consist of three identical beam splitters arranged in such a way that each beam undergoes identical reflections and transmissions on passing through the system (Fig. 65n).

**Achromatic Beam Splitters**

These devices are introduced into an incident beam of radiation when it is desired to divide it into two beams of approximately equal relative spectral composition but propagating in two different directions. In neutral beam splitters, the quantity $0.5(R_p + R_s)\theta_{\text{inc}}$ is always close to the reflectance at normal incidence, even though the individual $R_p$ and $R_s$ values may be quite different. The reflectance of absorbing, uncemented beam splitters depends also on the direction of incidence (see "Matrix Theory for the Analysis of Multilayer Systems in Sec. 42.3"). The optimum values of $T$ and $R$ depend on the application. For example, for a binocular eyepiece on a nonpolarizing microscope, the most important requirement is $T_p = T_s = R_p + R_s$ (Fig. 66a). For a vertical illuminator ($R_p T_p + R_s T_s$) should be a maximum (Fig. 66b). The condition for maximum fringe contrast in some interferometers requires that $R_1, T_p = R_2, T_p$ and $R_1, T_s = R_2, T_s$ (Fig. 66c). This is satisfied automatically by all nonabsorbing and by absorbing cemented beam splitters. The occasional requirement that the phase change on reflection be the same for radiation incident onto the beam splitter from opposite sides is automatically

![FIGURE 66 Three different ways of using beam splitters.](image-url)
FIGURE 67 Measured spectral transmittance and reflectance for polarized and unpolarized light of (a) Inconel- and (c) dielectric-coated beam-splitting plates (Oriel) and of (b) silver- and (d) dielectric-coated beam-splitting cubes (after Anders).

satisfied at the design wavelength by all-dielectric coatings composed of \( \lambda/4 \) layers, but not by uncemented metal beam splitters.\(^{196}\)

For maximum efficiency with unpolarized radiation, \( R_p, R_s, T_p \), and \( T_s \) should all approach 0.5. However, such a coating will not necessarily exhibit the best ratio of the intensities of the directly transmitted or reflected radiation to that which first undergoes multiple reflections.\(^{195}\) Often, beam splitters are required that are uniform over a broad spectral region. Inconel films satisfy this requirement although about one-third of the incident radiation is lost through absorption (see also “Neutral Filters” in Sec. 42.10). The design of achromatic all-dielectric beam splitters has been discussed by many workers.\(^{197-202}\) Knittl considered the design of beam splitters in which both the reflectance and the phase change on reflection are achromatized.\(^{203}\) The measured performance of several beam splitters is shown in Figs. 67 and 68.

Beam splitters for the x-ray region described so far operate at close to normal incidence and are effective only over a very narrow range of wavelengths. They consist of multilayer reflecting stacks (see “Multilayer Reflectors Made of Absorbing Materials”) in Sec. 42.7 and “Multilayer Reflectors for the Soft X-Ray and XUV Regions” in Sec. 42.8) deposited onto membranes or onto substrates that are thinned to enhance the transmitted component (Fig. 69).\(^{204}\)

**Nonpolarizing Beam Splitters.** For some applications it is important that the beam splitter introduce no polarization effects. Azzam has shown that, with the appropriate single layer on the face of a suitable high-refractive index prism, it is possible to construct a polarization-independent beam splitter.\(^{205}\) This device is quite achromatic and, in addition, by changing the angle of incidence of the beam on the prism, the beam-splitting ratio can be tuned over a wide range of values (Fig. 70b). The principle of frustrated total internal reflection can also be used to design beam splitters that have a very good performance.\(^{206,207}\) In these devices, radiation is incident at a very oblique angle onto an air gap or low-refractive-index films at the interface between two prisms (Fig. 65c,f). Unfortunately, the performance of such systems is very sensitive to the angle of incidence (Fig. 70c).

In many applications, it is important that the beam splitter be relatively insensitive to
FIGURE 68 Measured performance of a 45° infrared beam splitter consisting of a suitably coated ZnSe plate. (After Pellicori.455)

FIGURE 69 Measured performance of an x-ray beam splitter consisting of 11 pairs of Mo and Si layers on an 0.03-μm-thick Si₃N₄ membrane, operating at an angle of incidence of 0.5°. (After Ceglio.439)

FIGURE 70 Calculated performance at three angles of incidence of beam splitters with an incident medium of air: (a) beam splitter of the type glass (HL)² air, where \( n_H = 2.35 \), \( n_L = 1.38 \); (b) single layer \( n = 1.533, d = 0.1356 \) μm on a prism \( n_s = 2.35 \) (after Azzam425); (c) 15-layer frustrated-total-internal-reflection beam splitter (after Macleod207). The second diagram in each row corresponds to the performance at the design angle. The last two systems are fairly polarization-independent and yield different T/R ratios for different angles of incidence.
FIGURE 71 Calculated performance of polarization-insensitive achromatic beam splitters consisting of layer systems cemented between glass prisms: (a) all-dielectric system of the type $n_5$ (LMHMHML) $n_6$, where $n_r = 1.52$, $n_g = 1.38$, $n_H = 1.63$, $n_M = 2.35$. The quarter-wave layers are matched for 45° incidence (After Thelen). (b) three-layer metal/dielectric system (after Chiang). (c) polarization-independent beam-splitting arrangement of the type of Fig. 65 based on dielectric-metal-dielectric layer systems embedded between two prisms have been described. Much work has been done to find solutions based on dielectric layers only. However, the improvement is frequently at the expense of the width of the spectral region over which the beam splitter is effective. Some typical results are shown in Figs 71 and 72.

FIGURE 72 Measured performance of polarization-insensitive beam splitters of the type of Fig. 71 produced for three different spectral regions. (After Konoplev.)
Simple angle- and polarization-insensitive mechanical solutions to the achromatic beam-splitting problem exist if the application can tolerate spatial or temporal beam sharing (Fig. 73).

Color-selective Beam Splitters

For various technological applications, a beam of light must be divided into several components of different color. All-dielectric color-selective beam splitters (dichroics) are used for this purpose because they are practically lossless and because their transition wavelengths can be selected at will. They are essentially cut-off filters (see Sec. 42.9) usually designed for use at 45° incidence. Their spectral characteristics normally depend on the polarization of the incident radiation. The effect of this and of the variations in the angle of incidence and thickness of the coatings on the chromaticity coordinates of dichroic beam splitters for television cameras was investigated by Pohlack.\textsuperscript{219} If necessary, the polarization of the derived beams can be reduced through the use of auxiliary normal-incidence cut-off filters.\textsuperscript{220} Typical transmittance curves of several color-selective beam splitters are shown in Fig. 74.

Nonpolarizing Edge and Bandpass Filters. For more exacting applications, such as for use in multiplexers and demultiplexers, or for the separation of emission or absorption lines in atmospheric physics or Raman spectroscopy, it is possible to design and construct short- and long-wavelength color-selective beam splitters in which the polarization splitting has been largely eliminated.\textsuperscript{18,221} In the designs described, the polarization splitting is usually removed for all angles smaller than the design angle, but the cut-off wavelength
still shifts with the angle of incidence (Fig. 75). Some of the designs do not have a wide transmission region.

Neutral Filters

These devices are used whenever the intensity of the incident radiation is to be reduced uniformly throughout an extended part of the spectrum. The radiation usually traverses neutral filters at or near normal incidence. A number of absorbing glasses and gelatin filters are suitable for making neutral-density filters with densities of up to 5.0. However, their spectral transmission curves are not very uniform.

Evaporated films of metals such as aluminium, chromium, palladium, platinum, rhodium, tungsten, and alloys such as Chromel, Nichrome, and Inconel have been used for a long time to produce filters with densities of up to 6.0. A disadvantage of such filters is their high specular reflection. At present Inconel is commonly used for high-precision neutral-density filters. Chromium is favoured when tough, unprotected coatings are required (Fig. 76). The operating range and neutrality depends on the substrate materials.

**FIGURE 74** Measured spectral transmittance of four color-selective beam splitters. (Optical Coating Laboratory.)

**FIGURE 75** Calculated reflectance of a polarization-independent color-selective beam splitter. (After Thelen.)

**FIGURE 76** (a), (b) Spectral transmittance of various neutral density materials. Curve 1: tungsten film on glass; curves 2 and 3: diffuse and specular transmittance of photographic emulsion; curve 4: M-type carbon suspension in gelatine; curve 5: Inconel film on glass; curve 6: photographic silver density; curve 7: Wratten 96 density filter; curve 8: chromium film; curves 9 and 10: Chromel A film on glass evaporated at pressures of $10^{-2}$ and $10^{-4}$ Torr, respectively. (Curves 1 to 3, 9, and 10 after Banning; curves 4 to 7, Eastman Kodak Company; curve 8, Optical Coating Laboratory.)
The spectral-transmittance curves of neutral-density filters on magnesium fluoride, calcium fluoride, quartz, glass, sapphire, and germanium substrates are shown in Fig. 77. Linear and circular metal-film neutral-density wedges and step filters are also available commercially.

At times, there may be a need for a neutral attenuation that is not based on absorption. Sets of all-dielectric multilayer coating designs have been published with uniform transmission levels for the ultraviolet, visible, and near-infrared parts of the spectrum (Fig. 78).

### 42.11 INTERFERENCE POLARIZERS AND POLARIZING BEAMSplitters

The dependence of the optical properties of thin-film systems on the plane of polarization of obliquely incident radiation can be exploited to design interference polarizers and polarizing beam splitters with properties that augment those attainable by other means. The main difference between a polarizer and a polarizing beam splitter is that in the former only one polarized beam is required, whereas in the latter both beams are to be utilized. A polarizing beam splitter can therefore also be used as a...
polarizer. The performance in transmission or reflection of both devices is usually characterized by their degree of polarization $P$, 

$$P = \frac{T_p - T_s}{T_p + T_s} \quad \text{or} \quad P = \frac{R_p - R_s}{R_p + R_s}$$ (59)

In a polarizing beam splitter, a high degree of polarization is required in both beams and this is more difficult to achieve. Presently, efficient interference polarizers and polarizing beam splitters can be constructed for the ultraviolet, visible, and infrared spectral regions, and they are of particular interest whenever large areas and low losses are required. Schematic representations of the geometries of some of the devices are given in Fig. 65.

**Multicomponent Polarizers**

It is always possible to find an angle of incidence at which a substrate coated with a film of quarter-wave effective thickness will reflect only radiation polarized perpendicular to the plane of incidence. This property can be used to construct efficient transmitting polarizers using far fewer plates than necessary in the conventional pile-of-plates polarizer of equal performance (Fig. 65d). The calculated degree of polarization attainable with different numbers of plates is shown in Fig. 79 for a series of film indexes and polarizing angles. Experimental results agree closely with the calculations. In polarizers of this type, the variation of the degree of polarization is small over a wavelength span of one octave and for angular apertures of up to $\pm 10^\circ$. Yet, because of its bulk, this type of polarizer is not frequently used. An exception are polarizers for the infrared, where it is more difficult to produce multilayers composed of many layers. Because of the high refractive indices available in that spectral region, it is possible to achieve a high degree of polarization even after a single transmission through a plate coated with one layer only.

Several geometries of the reflection equivalent of the multiple-plate polarizer exist (Fig. 65). The reflectors can consist of one or more dielectric layers deposited onto nonabsorbing parallel plates or prisms (Fig. 65s to v). In other devices, the substrates are made of metal, or are coated with an opaque metallic film (Figs. 65o to r). The angles of incidence on the various mirrors need not be the same. Polarizers of this type are particularly useful in the vacuum ultraviolet and infrared spectral regions. The measured performances in the XUV region of two polarizers that are based on three reflections are depicted in Fig. 80. Interesting variants are polarizers that are based on total internal reflection or frustrated total internal reflection (Figs. 65l and m).
Polarizers for the soft x-ray region can be based on the fact that the reflectance of an x-ray multilayer mirror at oblique angles is very different for radiation polarized parallel and perpendicular to the plane of incidence. Because the region of high reflection is very narrow, multilayer x-ray polarizers essentially operate at one wavelength only. However, if the radiation is reflected from two identical mirrors (Fig. 65p), it is possible to construct a device with a reasonable throughput (>0.05) that can be tuned over a wide range of wavelengths without changing the direction of the emerging beam (Fig. 81).
Plate Polarizers

The number of coated plates in the transmission polarizers described here can be reduced without compromising the performance by depositing more than one high-refractive-index layer onto the surface of a plate and by spacing them with low-index films. In particular, it is possible to minimize the surface scatter, plate absorption, and lateral beam displacement by combining all the layers into one coating.

The most common solution for high-power laser applications is plate polarizers that are based on the polarization splitting that occurs at higher angles, for example, at the edges of quarter-wave stack reflectors (Fig. 65b). The plate on which the multilayer is deposited is held at Brewster’s angle with respect to the incident light to avoid second surface reflections. Usually the long wavelength edge of the reflector is used and the design is somewhat modified to remove the ripples in the transmission band of the \( p \)-polarized radiation (Fig. 82). The use of other multilayer structures, such as bandpass filters, for the construction of plate polarizers has also been proposed. The wavelength range over which plate polarizers are effective is much smaller than that of polarizers based on a series of coated plates, but this is acceptable for most laser applications.

Other methods for the design of narrowband plate polarizers based on two or three coating materials have been described by Minkov, Mahlein, and Thelen. However, these solutions require many layers and sometimes very oblique angles of incidence.

Embedded Polarizers and Polarizing Beam Splitters

Polarizers and polarizing beam splitters effective over a wider spectral region are obtained when multilayer coatings of the type \([HL]^N\) or \([(0.5H)L(0.5H)]^N\) are embedded between media of higher refractive index. The higher the refractive-index ratio of the two coating materials used, the fewer the number of layers needed to achieve a certain degree of polarization and the wider the spectral region over which the polarizer will be effective. However, a certain relationship between the refractive indices of the materials and the angle of incidence must be satisfied. The optical thicknesses of the quarter-wave layers should be matched for the angle of incidence.

A particularly convenient polarizer with no lateral beam displacement results when the multilayer is embedded between two right-angled prisms, as shown in Fig. 65c. MacNeille polarizers operate over a very broad range of wavelengths (Figs. 83a and b). For best results, the following relationship between the angle of incidence \( \theta_p \) and the refractive indices of the prism and the layers should be satisfied:

\[
 n_p \sin \theta_p = \frac{n_L n_H}{(n_L^2 + n_H^2)^{1/2}}
\]

(60)
The measured degree of polarization \( P \) and transmittance of parallel and crossed McNeille interference polarizers for the ultraviolet spectral region. (c) represents the results obtained when the cemented polarizers shown in (a) and (b) are placed in series (after Sokolova and Krylova\(^{465}\)); (d) is the measured performance of an optically contacted polarizer with a high laser damage threshold (after Wimperis\(^{466}\)).

This expression is independent of the thicknesses of the layers and, were it not for the dispersion of the optical constants, the transmission for the \( p \) polarization would be 1.0 across the entire spectral region. However, the dispersion of the high-index material will tend to decrease the useful spectral range of the polarizer.\(^{242}\) It is possible to select the \( V \)-number of the substrate material in order to decrease this disturbing effect. The rejection of the unwanted polarization will normally not be the same in both beams, although this can be achieved at the expense of the wavelength range.\(^{243}\) If polarizers for an even wider spectral region are required, it is possible to extend the range by placing two polarizers in series (Fig. 83c), by using the technique of superposition of stacks with contiguous high-reflectance zones (see “All-Dielectric Broadband Reflectors” in Sec. 42.8) or of two periodic multilayers with thickness ratios of 1:1 and 1:2.\(^{156}\) When both beams are used, the useful angular field of the MacNeille polarizer is of the order of \( \pm 2^\circ \). It is possible to increase it to \( \pm 10^\circ \), but again at the expense of the spectral range.\(^{244}\)

When the MacNeille polarizer is to be used as a polarizing beam splitter, it is possible to design it for normal incidence of the beams onto the prism faces, or for a 90° deflection between the two beams (Fig. 65j).\(^{245,246}\) However, it is more convenient to embed the multilayers between two 45° prisms.\(^{247}\) Another way of avoiding the use of a cement is to optically contact the polarizer prisms (Figs. 65g, 83d).\(^{248}\) More frequently, the layers are deposited onto the hypotenuse of one or two air-spaced prisms (Fig. 65e).\(^{249}\) However, this arrangement is similar to a plate polarizer designed for use at 45° and so is its performance.

In Fig. 86a to h the calculated spectral and angular performances of a number of
polarizers and polarizing beam splitters are compared. A theoretical comparison of the properties of the MacNeille, cube, and plate polarizers for one wavelength has been given by Cojocaru.\textsuperscript{250}

### 42.12 BANDPASS FILTERS

An ideal bandpass filter transmits all the incident radiation in one spectral region and rejects all the other radiation. Such a filter is completely described by the width of the transmission region and the wavelength at which it is centered. Practical filters are not
FIGURE 86 Calculated spectral and angular performance of several types of polarizers and polarizing beam splitters with an air incident medium. Calculations assume that all nonmetals are absorption- and dispersion-free. (a) Multiple plate polarizer; (b) single-reflection polarizer (after Azzam); (c) three-reflection polarizer (after Thonn); (d) plate polarizer (after Songer).

perfect and require more parameters to adequately describe their performance. No uniform terminology has yet been developed for this purpose. Care should be taken when reading and writing specifications since often different terms are used to describe different types of filters, and sometimes quantities bearing the same name are defined differently.

The position of the transmission band is variously specified by the wavelength \( \lambda_{\text{max}} \) at which the maximum transmission occurs, the wavelength \( \lambda_0 \) about which the filter
passband is symmetrical, or the spectral center of gravity \( \lambda_c \) of the band. When specifying the tolerance on \( \lambda_c \) it should be remembered that the peak of interference filters can be moved only towards shorter wavelengths by tilting (see “Matrix Theory for the Analysis of Multilayer Systems” in Sec. 42.3).

The peak transmittance \( T_p \) may or may not take into account the absorption within the
substrate and/or blocking filters used to remove the unwanted transmission of the interference filter away from the principal passband (Fig. 87).

The half-width (HW) \( \Delta \lambda_{0.5} \) of the filter is the difference between the wavelengths at which the transmittance is a half of \( T_0 \). This quantity is also sometimes called the full-width half-maximum (FWHM). It is often expressed as a percentage of \( \lambda_o \). The base-width (BW) \( \Delta \lambda_{0.01} \) is similarly defined. The ratio \( \Delta \lambda_{0.01}/\Delta \lambda_{0.5} \), sometimes called the shape factor, indicates how "square" the transmission band is. Sometimes widths corresponding to other fractions of the transmittance are used to define it.

The minimum transmittance \( T_{\text{min}} \) of the filter does not take into account the effect of blocking filters. The quantity \( T_{\text{min}}/T_0 \) is called the rejection ratio.

In all-dielectric transmission-band filters, the transmittance rises at some distance on either side of the transmission band. The distance over which the transmittance is low is called the rejection region. The distance between the two transmission maxima adjacent to the principal transmission band is called free spectral range. Should either of these two quantities be inadequate, auxiliary blocking filters might have to be provided.

The ultimate measure of the suitability of a bandpass filter with a blocked spectral transmittance \( T(\lambda) \) for a particular application is the signal-to-noise ratio SN, defined in terms of the spectral energy distribution \( I(\lambda) \) of the source and the spectral detectivity \( D(\lambda) \) of the detector,

\[
SN = \frac{\int_{\lambda_1}^{\lambda_2} I(\lambda)T(\lambda)D(\lambda) \, d\lambda}{\int_{0}^{\lambda_1} I(\lambda)T(\lambda)D(\lambda) \, d\lambda + \int_{\lambda_2}^{\infty} I(\lambda)T(\lambda)D(\lambda) \, d\lambda}
\]

where \( \lambda_1, \lambda_2 \) are the lower and upper limits of the transmission region of the filter. The SN ratio is sometimes expressed in terms of optical density.

Useful general reviews of bandpass filters exist.\textsuperscript{2,251,252} Interference filters, and especially bandpass filters, are increasingly deposited in complicated millimeter and submillimeter patterns for use with display devices and detectors.\textsuperscript{253,254} Very fine masks or photolithographic processes are required to produce such structures (Fig. 88).

\[\text{FIGURE 87} \quad \text{Definition of some of the terms used to describe the properties of narrow-bandpass filters (see section, "Bandpass Filters"). The curve represents the measured transmittance of an unblocked second-order metal-dielectric interference filter of the Fabry-Perot type (see "Filters with Metallic Reflecting Coatings"). (After Bausch & Lomb.)}\]

\[\text{FIGURE 88} \quad \text{Four-color checkerboard pattern for use with a 64 x 64 element focal plane HgCdTe detector array. The dimensions of each element are 100 x 100 \text{µm}. (Reproduced with permission from Barr Associates.)}\]
FIGURE 89  Schematic representations of various types of bandpass interference filters: (a) to (c) Fabry-Perot interference filters with metal, dielectric, and metal-dielectric reflectors; (d) frustrated-total-internal-reflection filter; (e) and (f) square-top multicavity filters with metal and dielectric reflectors; (g) induced-transmission filter; (h) phase-dispersion (spacerless) interference filter; (i) Fabry-Perot filter with a mica or quartz spacer.

Narrow- and Medium-bandpass filters (0.1 to 35 percent HW)

Even though the essential components of Fabry-Perot (FP) interference filters, i.e., the spacer and the two reflectors that surround it, can take on many different forms (see Fig. 89), the filters are essentially low-order FP interferometers and hence the theory developed for the latter (see, for example, Born and Wolf\(^2\)) applies in full.

The transmittance of an FP-type filter, not allowing for absorption and multiple reflections within the substrate, is given by

\[
T = \frac{T_R^2}{(1 - R)^2 + 4R \sin^2 \delta} \tag{62}
\]

where

\[
T_R = \sqrt{T_1 T_2}, \quad R = \sqrt{R_1 R_2} \tag{63}
\]

\(T_1, R_1\) and \(T_2, R_2\) are the transmittances and reflectances of the first and second reflectors, respectively, as seen from within the spacer medium, and \(\delta\) is given by

\[
\delta = \frac{2\pi}{\lambda} nd \cos \theta + \varepsilon \tag{64}
\]

\[
\varepsilon = \frac{\varepsilon_1 + \varepsilon_2}{2} \tag{65}
\]

\(n, d,\) and \(\phi\) are the refractive index, thickness, and angle of refraction of the spacer,
respectively, and \( \varepsilon_1 \) and \( \varepsilon_2 \) are the phase changes on reflection from the spacer side of the first and second reflectors at a wavelength \( \lambda \). Maxima of \( T \) occurs at wavelengths.

\[
\lambda_n = \frac{2nd \cos \theta}{k - \varepsilon/\pi} \quad k = 0, 1, 2 \ldots
\]

and are given by

\[
T_n = \left( \frac{T_R}{1-R} \right)^2 = \left( \frac{1}{1+A/T_R} \right)^2
\]

\( A(=1-T_R-R) \) is the mean absorption of the reflectors. The minimum transmittance

\[
T_{\text{min}} = \left( \frac{T_R}{1+R} \right)^2
\]

occurs at

\[
\lambda_{\text{min}} = \frac{2nd \cos \theta}{k - \varepsilon/\pi} \quad k = \frac{1}{2}, \frac{3}{2}, \frac{5}{2} \ldots
\]

If \( T_R \) and \( R \) are essentially the same at \( \lambda_o \) and \( \lambda_{\text{min}} \), the rejection ratio is given by

\[
\frac{T_{\text{min}}}{T_n} = \left( \frac{1-R^2}{1+R} \right)
\]

For \( R > 0.7 \) the half-width of the transmission band (expressed as a percentage of \( \lambda_o \)) is given by

\[
\frac{\Delta \lambda_{0.5}}{\lambda_o} \times 100 = \frac{1-R}{\sqrt{R}} \frac{100}{2\pi nd \cos \theta} \frac{\partial \varepsilon}{\partial \lambda}
\]

For a given order of interference the half-width and rejection ratio cannot be varied independently. The formula

\[
T = \frac{T_n}{1 + 4(\lambda - \lambda_o)/\Delta \lambda_{0.5}}
\]

valid for FP filters with small values of \( \partial \varepsilon / \partial \lambda \) in the neighborhood of \( \lambda_o \), represents a Lorentzian line shape with \( \Delta \lambda_{0.5} = 3\Delta \lambda_{0.1} \) and \( \Delta \lambda_{0.1} = 10\Delta \lambda_{0.5} \). The shape factor of all FP-type interference filters is therefore of the order of 10.

Southwell has recently shown that a spacer in an interference filter need not consist of a single layer only, and that there are some advantages when it is partitioned into a number of layers.  

---

**Fabry-Perot Interference Filters (0.1 to 10 percent HW)**

*Filters with Metallic Reflecting Coatings.* This is the first interference bandpass filter ever made, and the simplest. It consists of two partially transmitting, highly reflecting metal layers separated by a dielectric film and is symbolically represented by MDM (Fig. 89a). The best metallic reflectors currently available are aluminium and silver for the 0.125- to 0.34- \( \mu \)m and 0.34- to 3.0- \( \mu \)m spectral ranges, respectively. The measured spectral-transmittance curves of a number of typical filters are shown in Fig. 90. The phase changes on reflection at the spacer-metal-reflector surface are finite and hence they affect the position of the transmission maxima [Eq. (66)]. But the dispersion of
the phase change on reflection can be neglected, and so the half-width depends only on the
reflectance of the metal layers and the order of the spacer. Filters with half-widths of 1 to 8
percent are common. Because the ratio A/T is not very small for metals, the maximum
transmission is limited [Eq. (67)]. Maximum transmittances of 40 percent are relatively
common. For filters with the narrower half-widths, or for shorter wavelengths, transmis-
tances of the order of 20 percent have to be accepted. This is much less than the
transmittances of all-dielectric filters of comparable half-widths. Nevertheless, filters of this
type are useful because their transmittances remain low except at wavelengths at which the
first- and higher-order transmission maxima occur [Eq. (66)]. Blocking is thus easy. In
particular, first-order filters usually do not require any blocking on the high-wavelength
side—a difficult task at all times. The rejection of the filters is good, though not
spectacular. If a better rejection is required and a lower transmittance can be tolerated,
two identical filters may be cemented together (curve 6, Fig. 90). This is possible because
of the finite absorption in the metal films. Alternatively, metal square-top filters or filters
with even more complicated structures can be used (see Sec. 42.12).

Filters with All-dielectric Reflectors. Above 0.2 μm, the metallic reflectors can be
replaced by all-dielectric quarter-wave stacks (see Sec. 42.7). The symbolic representa-
tion of such a filter (Fig. 89b) is, for example, [HL]^m[H][LH]^n or [H][LH]^2m[L][HL]^2H,
H and L being quarter-wavelength layers of high and low refractive indices, respectively;
m is the order of the spacer and N the number of full periods in the reflecting stacks. The
phase change on reflection at the boundary between the spacer and such a reflector does
not affect the position of \( \lambda_o \) (Eq. (66)) but the dispersion of the phase change on reflection
is finite, depends on the materials used, and for lower-order spacers, contributes very
significantly to the reduction of the half-width of the transmission band [Eq. (71)]. Expressions for the half-width

\[
\frac{\Delta \lambda_{0, \text{L}}}{\lambda_{0, \text{L}}} \times 100 = \frac{4n_h n_L^{2n}(n_H - n_L) \times 100}{m \pi n_\text{H}^{N+1}(n_H - n_L + n_L/m)}
\]

\[
= \frac{4n_h n_L^{2n-1}(n_H - n_L) \times 100}{m \pi n_\text{H}^{N}(n_H - n_L + n_L/m)}
\]

(73)

for high- and low-refractive index spacers, respectively have been given by Macleod.2

![Figure 90](image_url)
By choosing a suitable combination of the reflectance and order of the spacer almost any half-width between 0.1 and 5 percent can be achieved in the visible part of the spectrum while maintaining a useful rejection ratio.

The maximum transmittances of all-dielectric FP filters depart from unity because of the finite absorption, scattering, and errors in the thicknesses and refractive indexes of the films. In the central part of the visible spectrum, maximum transmittances of 0.8 are normal for unblocked filters with a half-width of 1 percent, although higher transmittances can be achieved. This figure is gradually reduced as \( \lambda \) approaches 0.2 or 20 \( \mu m \), and filters with narrower half-widths become impracticable for lack of adequate transmittance.

The transmittance away from the transmission maximum is low only over the extent of the rejection region of the two materials used for the construction of the reflectors (Fig. 32), and additional blocking is often required on both the long- and the short-wavelength sides. This can result in a considerable lowering of the maximum transmittance of the blocked filter, a 30 to 40 percent loss being not uncommon for filters peaked in the ultraviolet or infrared spectral regions.

For those parts of the visible and infrared for which nonabsorbing mechanically robust coating materials abound, square-top interference filters (see "Square-Top Multicavity, Bandpass Filters" in Sec. 42.12) are often preferred because of their better shape factor and higher rejection ratio. All-dielectric FP filters are still attractive in the ultraviolet, where there is a lack of such materials and where thickness monitoring is difficult, and also in the far-infrared, where very thick layers are required.

The measured transmittance curves of a number of typical all-dielectric FP filters are shown later in Fig. 94. An additional curve on a smaller scale is given, whenever necessary, to show the transmittance away from the passband.

**Filters with Metal-dielectric Reflectors.** In these filters the reflectors consist of metal layers whose reflectance has been enhanced through the addition of several dielectric layers (see "Enhancement of Reflection" in Sec. 42.16) (Fig. 89c). The properties of such filters are intermediate to those described in the two previous sections.

**Frustrated-Total-Internal-Reflection Filters.** These are essentially FP filters in which the spacer layer is surrounded by two frustrated-total-reflection surfaces (Fig. 89d). They have not found wide applications as bandpass filters because the finite absorption and scattering within the layers have prevented the theoretically expected high transmittance and small half-widths from being realized and because the angular variation of the wavelength of the transmission peaks is very high.

**Square-top Multicavity Bandpass Filters (0.1 to 35 percent HW).** A filter with a "squarer" shape that does not suffer from some of the disadvantages of the FP filters results when the basic FP structure is repeated two or more times. Such filters may be based on metal\(^{257} \) or all-dielectric\(^{262,263} \) reflectors (Fig. 89e and f). Thus, for example, \([HL]^{n/2}H[LH]^{n} \) represents an all-dielectric square-top filter in which the FP structure \([HL]^{n/2}H[LH]^{n} \) is repeated twice. The quarter-wavelength layer C is called a coupling or tie layer, and the half-wavelength-thick spacer layers \( 2H \) are often called cavities. Small departures from this model are made at times to improve the transmittance in the pass band or the angular properties of the filter.

The half-widths of the narrower multicavity filters of the preceding type do not differ very significantly from those of the basic FP structure [Eq. (71)]. The shape factors decrease with an increase in the number of the cavities and do not seem to depend on the materials.\(^{258} \) They are approximately 11, 3.5, 2.0, and 1.5 for one, two, three, and four cavities, respectively. The minimum transmittance in the rejection region is roughly that which could be obtained if the filter were composed entirely of \( \lambda/4 \) layers [Eq. (44)]. Unlike in the FP filter, there is therefore some independent control of the half-width and rejection ratio. These various points are illustrated in Fig. 91. The peak transmittance of multicavity square-top filters is less affected by the residual absorption in the layers than
that of FP-type filters. As in the case of their FP counterparts, metal-dielectric square-top filters can be cemented together to enhance the rejection (curve 5, Fig. 92).

The improvement in the performance of square-top bandpass filters over that of the FP type is such that, despite their more critical and expensive production, most manufacturers regard them as their standard line of filters. The spectral-transmittance characteristics of typical commercially produced metal-dielectric and all-dielectric bandpass interference filters of different half-widths are shown in Figs. 92 and 95 to 98, respectively. Filters with intermediate half-widths and peak wavelengths can readily be obtained.

For very critical or special applications, multicavity filters are designed and constructed with properties that exceed those shown in the preceding figures. For example, for the use in fiber-optic communications systems, multicavity filters are required in which the peak transmittance closely approaches unity. Various procedures for the design of such filters, including some that are based on the use of Chebyshev polynomials, have been described. Special care has to be taken during the manufacture of the coatings to meet this requirement. Typical measured spectral transmittance curves are shown in Fig. 99. For other applications, such as fluorescence or Raman spectroscopy, the peak

**FIGURE 91** Calculated transmittance on a logarithmic scale of the bandpass filters: air-
\[n_1(n_2(0.5,0.5))^{N-1}(0.5,0.5)]^{-1} \text{glass}, N = 1, 2, \text{and } 3.

**FIGURE 92** Measured transmittance of square-top bandpass filters with metallic reflecting coatings. Curve 5 corresponds to the transmission of two identical filters cemented together. (Curves 1 and 2, Schröder; curves 3 and 6, Balzers; curves 4, 5, and 7, Schott & Gen.)
FIGURE 93 Measured transmittance of very narrow bandpass interference filters with half-widths less than 0.1 percent. Evaporated spacers: (a) (after Meltzer\textsuperscript{476}); (b) (after Eather and Reason\textsuperscript{276}); (c) mica interference filter for $H_{\alpha}$; (d) mica interference filter with transmission bands polarized at right angles to one another (Heliotek\textsuperscript{448}); (e) single and (f) and (g) double quartz-spacer interference filters (after Austin\textsuperscript{287}). The dotted curves in Figs. 93 and 94 represent the transmittances of the filters plotted over a ten-times-wider spectral region.

FIGURE 94 Measured transmittance of FP all-dielectric interference filters with narrow half-widths. Evaporated spacers: (a) (after Cohendet and Saudreau\textsuperscript{477}); (b) and (d) (after Motoyilo\textsuperscript{478}); (c) and (e) (after Neilson and Ring\textsuperscript{479}); (f) (after Turner and Walsh\textsuperscript{480}); (g) (after Smith and Seely\textsuperscript{481}). The dashed curves in Figs. 94 and 95 correspond to a transmission range of 0.0 to 0.1.
FIGURE 95 Measured transmittances of fully blocked square-top interference filters with half-widths between 0.25 and 1.4 percent. (a), (c), and (f) Corion; (b) (after Bliford); (d) (Helios); (e) (Baird Atomic); (g) is the only filter in the series that is not blocked (after Smith and Seely).

FIGURE 96 Measured transmittances of fully blocked square-top interference filters with half-widths of the order of 5 percent. (Curves 1 and 2, Corion; curve 3, Spectrum Systems; curves 4 and 6, after Turner; curve 5, Eastman Kodak; curves 7 and 8, Optical Coating Laboratory.)
FIGURE 97 Measured transmittances of fully blocked square-top interference filters with half-widths of the order of 10 percent. (Curve 1, Bausch and Lomb; curve 2, Baird Atomic; curve 3, Infrared Industries; curve 4, after Turner; curve 5, Eastman Kodak; curve 6, Optical Coating Laboratory.)

FIGURE 98 Measured transmittances of blocked square-top interference filters with half-widths of the order of 25 percent. (Curve 1, Heliotek; curves 2 and 4, Optical Coating Laboratory; curves 3 and 6, after Turner and Walsh; curve 5, Infrared Industries.)
transmittance is not important, but signal-to-noise ratios of the order of $10^{-8}$ are required. This necessitates the use of many cavities (Fig. 100).

**Induced-transmission Filters.** The transmittance of a metal layer can be considerably enhanced by surrounding it with suitable multilayer structures (Fig. 89g). Thus, for example, it is possible to induce a transmittance of 65 percent at $\lambda = 0.25\,\mu m$ in a 0.03-μm-thick aluminum film which, when deposited directly onto a quartz substrate, would transmit only 2.5 percent of the same radiation.\textsuperscript{266} The induced transmittance is highly wavelength-sensitive and can be used to construct bandpass filters containing one or more metal layers.\textsuperscript{267–274} Induced-transmission filters combine the good long-wavelength attenuation properties of the more common types of metal/dielectric filters (earlier in this section) with peak transmittances that are closer to those of all-dielectric filters. The performances of some experimentally produced induced-transmission filters are shown in Fig. 101.

**Very Narrow Bandpass Filters (HW—0.1 percent)**

It follows from Eq. (71) that the half-widths of interference filters can be reduced by increasing the reflectance of the reflectors, the order of interference of the spacer, or the dispersion of the phase change on reflection. All these approaches have been tried in the past.

**FIGURE 99** Calculated and experimental spectral transmittance and measured attenuation curves of (a) 3-, (b) 4-, and (c) 5-cavity bandpass filters. (After Minowa\textsuperscript{485})

**FIGURE 100** Measured spectral transmission characteristics of two bandpass filters for fluorescence applications and of a cut-off filter for Raman spectroscopy. (After Omega Optical, Inc.)\textsuperscript{432}

**FIGURE 101** Measured transmittance of induced transmission filters for the ultraviolet, visible, and infrared spectral regions. (a) (after Tsypin\textsuperscript{486}); (b) (after Berning and Turner\textsuperscript{267}); (c) (after Holloway and Lissberger\textsuperscript{270}).
Filters with Evaporated Spacers (HW > 0.03 percent). In narrowband filters of conventional construction, both high-reflectance and higher-order spacers are used. The manufacturing process is quite critical, and attention must be paid to details. The films have to be very uniform over the filter area, and they must not absorb or scatter. They must not age, or, alternatively, their ageing must be capable of being accelerated or arrested. Monitoring must be precise, so that the peak occurs at or close to the desired wavelength.

Both FP and square-top filters of very narrow bandwidths can be made, the latter being preferable for most applications. The limit on the half-widths of this type of filter seems to be of the order of 0.03 percent.275 The performance of two commercially produced filters of this type are shown in Fig. 93a and b.

Fabry-Perot Filters with Solid Spacers. In practice, the half-width of interference filters cannot be reduced indefinitely by increasing the optical thickness of an evaporated spacer [Eq. (71)] because when the latter exceeds about two wavelengths, it may become too rough to be useful.276 A high-order filter can, however, be constructed by evaporating reflecting coatings on either side of a thin prefabricated spacer (Fig. 89i).

Mica Spacers (HW > 0.01 percent). Transmission bands in silvered mica were probably first observed by Wood,277 but the deliberate use of mica to construct filters came much later.278–281 The construction of mica interference filters with transmittances of 30 to 80 percent per polarization for half-widths of the order of 0.01 to 0.1 percent in the 0.45- to 2.0-μm wavelength region is relatively straightforward.280 The position of the transmission peak can be located within a fraction of an angstrom, does not change with time, and can be sufficiently uniform over areas of 2- to 5-cm diameter. Because of the very high order of interference (70 to 700 orders), the spectral free range is quite small, and auxiliary filtering is necessary for most applications. Unless the thickness of the mica is specially selected, the birefringence of mica will result in two mutually perpendicularly polarized sets of transmission bands, a fact that can be used to advantage in some applications. The spectral-transmittance curve of a fully blocked mica interference filter for Hα is shown in Fig. 93c.

Optically Polished Solid Spacers (HW > 0.002 percent). It is possible to construct very narrow bandpass filters having thin fused-quartz spacers.282–286 A good fused-quartz flat is coated with an all-dielectric reflector, and this coated surface is optically contacted to another flat.287,288 The flat is then ground down and polished to form a spacer layer of the required thickness, and the second reflector layers are applied to complete the filter. As in the mica filters, the position of the transmission band is very stable, and auxiliary blocking filters are needed because of the small free spectral range. The transmittance of filters with silica spacers is higher than that of corresponding mica filters because fused quartz is highly transparent and is not birefringent. A typical unblocked filter with a clear aperture of 3.5 cm and a half-width of 0.007 percent had a transmittance of 45 percent for nonpolarized light. An important advantage of filters with fused-silica spacers is that it has been found possible, by repeating the process described, to construct square-top filters with rejection ratios of the order of 5 × 10^4 (Fig. 93f) and filters with half-widths as low as 0.002 percent (Fig. 93g). However, such filters are very expensive.

Other materials can also be used to produce solid spacers by optical polishing. Germanium was used by Smith and Pidgeon290 and by Costich291 to produce very narrow bandpass filters for the infrared. Roche and Title used a substrate made of a combination of yttrium and thorium oxides to produce a filter with a HW = 0.004 percent at 3.3 μm.291

Plastic Spacers (HW > 0.15 percent). Mylar has very smooth surfaces and areas can be selected that have a sufficiently uniform thickness to permit the use of this material as a solid spacer. Candille and Saurel have used this material to produce narrow bandpass filters and obtained half-widths of the order of 0.0008 μm.292,293 In some of their designs, a second, evaporated narrowband filter deposited onto one side of the solid spacer served to remove unwanted adjacent orders.
Phase-dispersion Filters (HW > 0.1 percent). The dispersion of the phase change on reflection enters into Eq. (71) for the half-width of FP filters. Typical values of this quantity at λ = 0.5 μm for a silver reflector, a nine-layer quarter-wave stack with $n_H/n_L = 1.75$, and for a broadband reflector are −0.5, −6.8, and −112.0, respectively.104 As a result, the half-widths of FP filters constructed with such reflectors should be reduced by factors of about 1.05, 2, and 20. In the last case, the contribution of the spacer to the half-width is negligible, and a spacerless design is possible (Fig. 89h).294,295 Unfortunately, the expected reduction in half-width has so far not been fully realized in practice, probably because of errors in the monitoring and lack of uniformity of the layers.296

Tunable Filters (HW > 0.001 percent). These are usually air-spaced FP interferometers, often provided with elaborate automatic plate-parallelism and spacing control, which are more akin to spectrometers than to filters.297,298 The position of the passband can be tuned quite significantly by changing the separation between the reflector plates. This type of tuning, unlike the tuning of filters by tilting, does not affect the angular field or shape of the transmission band. Ramsey reviews the various problems associated with the construction and use of such instruments.299,300

An electrically tunable 0.005-percent half-width interference filter with a lithium niobate spacer sandwiched between two conducting reflecting coatings has also been described (Fig. 102).

Wide-bandpass Filters

Filters with half-widths ranging from 10 to 40 percent can be constructed using techniques described earlier in this section (Figs. 97 and 98). Filters with wider transmission bands are usually obtained by combining short- and long-pass filters with cutoffs at the desired wavelengths. The cut-off filters may be all-dielectric (Figs. 58 and 59), glass or gelatine cut-off filters, or antireflection-coated infrared materials. Some of the short- or long-pass filters may be regarded as being wide-bandpass filters in their own right. The cut-off filters may be combined into a single filter. Alternatively, it is possible to assemble a number of short- and long-wavelength cut-off filters into sets that make it possible to assemble wide-passband filters of different half-widths and peak wavelengths. In this latter arrangement, the cut-off positions of all-dielectric short- and long-pass filters can be tuned

![FIGURE 102 Performance of an electrically tunable narrowband filter with a lithium niobate spacer. (After Burton.487)]
individually by tilting to coincide with the desired wavelengths. Another way of tuning the edges of the transmission band is to pass the radiation through a pair of circular wedge short- and long-wavelength cut-off filters placed in series.

Filters with very broad transmission bands are also obtained when a multilayer is formed from two suitably displaced long-wavelength cut-off filters separated by an appropriate matching layer (Fig. 103a).\(^{301}\) Automatic optimization programs can be used to design high-transmission broadband filters with a high rejection and a shape factor close to unity (Fig. 103b).\(^{302}\)

**Interference Filters with Multiple Peaks**

For some applications, filters with multiple peaks in one particular spectral region are required. The design of such filters has been considered by Pelletier et al.\(^{303}\) Typical results are shown in Fig. 104. Filters with different peak separations, rejections, and half-widths are possible.
Linear and Circular Wedge Filters

If the thicknesses of all the layers of a bandpass filter vary in proportion across the surface of a substrate, the position of the transmission peak will vary in the same way (see “Matrix Theory for the Analysis of Multilayer Systems” in Sec. 42.3). Such wedge filters are as old as the interference filter itself and are available in versions in which the wavelength variation occurs along a straight line or a circle. The latter arrangement is particularly useful because it lends itself well to the construction of low-cost, small, and lightweight rapid-scan monochromators of moderate resolution that are robust and environmentally stable. Methods for the production of circular variable filters with a linear dependence of wavelength on angle and references to some of the previous work on wedge filters are given in several papers. Circular variable square-top filters for the 0.24- to 0.4- and for the 0.4- to 25-μm spectral regions are described by Avilov and by Yen. The maximum transmittances for fully blocked filters vary between 15 and 75 percent, depending on the spectral region and the half-width of the filter. Rejection levels of 0.1 or 0.01 percent are possible. Typical transmission curves for several angular positions on two circular variable square-top filters are shown in Fig. 105. The ratio of the maximum to minimum wavelength available in one wheel varies between 1.11 and 16.3. The angular width of the slit used in conjunction with a circular variable filter, expressed as a percentage of the angular size of the filter wedge, should not exceed the nominal half-width of the filter (expressed in percent) if it is not to cause a marked reduction in the resolution.

Angular Properties of Bandpass Interference Filters

With the gradual increase of the angle of incidence, the transmittance maximum of a typical bandpass filter moves towards shorter wavelengths. On further increase in the angle of incidence, the maximum transmittance and the half-width deteriorate; the transmission band becomes asymmetric and eventually splits up into p- and s-polarized components (Fig. 106a). The deterioration is more rapid for nonparallel radiation.

Properties of Bandpass Filters for Angles of Incidence Less than 20°. The behavior of bandpass filters for angles of incidence \( \theta_0 \approx 20^\circ \) can be described quantitatively using the concept of an effective index \( \mu^* \) of the filter. In terms of \( \mu^* \), the transmittance \( T \) in the neighborhood of the transmission peak of any FP filter is given by Lissberger:

\[
T = \frac{T_0}{1 + \left[ \frac{2(\lambda - \lambda_o)}{\Delta\lambda_{0.5}} + \frac{\lambda_o}{\Delta\lambda_{0.5}} \mu^* \theta_0^2 \right]^{\frac{\mu^*}{2}}}
\]  

(74)
FIGURE 106 Angular properties of all-dielectric interference filters: (a) measured variation with angle of incidence of the spectral transmittance of a typical commercial interference filter (after Bliford\textsuperscript{339}); (b) calculated transmittance of a filter in which the peaks of the two polarized transmission bands at nonnormal incidence coincide. The filter is of the type air-[HL]\textsuperscript{4}(2A)[LH]\textsuperscript{4}-glass, where \(n_H d_H = n_L d_L = n_A d_A = \lambda/4\) and \(n_r = 1.52, n_m = 1.00, n_A = 2.30, n_L = 1.38,\) and \(n_H = 1.825.\)

\(\Delta \lambda_{0.5}\) and \(T_o\) are the half-widths and the maximum transmittance (at \(\lambda_o\)) for normal incidence of the radiation. Formulas for \(\mu^*\) in terms of the construction parameters have been found for the all-dielectric FP filter and the double-spacer filter,\textsuperscript{313,314} and for the metal-dielectric FP and induced-transmission filters.\textsuperscript{315} The change in position of the transmission peak (\(\delta \lambda\)) and the half-width (\(\Delta \lambda_{0.5}\)) at angle \(\theta\) are

\[
\left( \frac{\delta \lambda}{\lambda_o} \right)_o = \frac{\theta_c^*}{2 \mu^*}
\]

(75)

and

\[
\frac{(\Delta \lambda_{0.5})_o}{\Delta \lambda_{0.5}} = \left[ 1 + \left( \frac{\theta_c^* \lambda_o}{\mu^* \Delta \lambda_{0.5}} \right)^2 \right]^{1/2}
\]

(76)

For convergent radiation of semiangle \(\alpha\), the corresponding expressions are

\[
\left( \frac{\delta \lambda}{\lambda_o} \right)_\alpha = -\frac{\alpha^2}{4 \mu^*}
\]

(77)

\[
\frac{(\Delta \lambda_{0.5})_\alpha}{\Delta \lambda_{0.5}} = \left[ 1 + \left( \frac{\alpha^2 \lambda_o}{2 \mu^* \Delta \lambda_{0.5}} \right)^{1/2} \right]^{1/2}
\]

(78)

Linder and Lissberger discuss the requirements and design of filters for this case.\textsuperscript{312,316}

Small tilts are commonly used to tune the peak of a filter to the desired wavelength even though they have an adverse effect on the angular field of the filter.

**Bandpass Filters with Little or No Polarization Splitting.** It has been shown numerically for the phase-dispersion filter,\textsuperscript{317} for the frustrated-total-reflection filter,\textsuperscript{318} and for the metal-dielectric\textsuperscript{23} and all-dielectric\textsuperscript{319} FP filters that it is possible to arrange for the two polarized transmission bands, which may have different widths, to coincide at high angles of incidence (Fig. 106b). The narrow, symmetrical high-transmittance bands that result may be useful for some applications even though the position of the maximum is still...
displaced with angle. Baumeister has shown how to design multicavity filters with no polarization splitting at one angle of incidence (Fig. 107).320

**Wide-angle Bandpass Filters.** Figure 108 shows the variation with effective index \( \mu^* \) of the angular field of FP filters, defined as being twice the angle of tilt necessary to reduce to 0.8\( T \), the transmittance of the filter for radiation of wavelength \( \lambda \). To increase the angular field, \( \mu^* \) must be increased. Thus, for example, in all-dielectric FP filter, the expression for \( \mu^* \) shows that \( n_L < \mu^* < n_H \) and that with increasing spacer order, \( \mu^* \) approaches the refractive index of the spacer (see also Ref. 321). The upper limits for \( \mu^* \) for an all-dielectric FP filter in the ultraviolet, visible, and infrared parts of the spectrum are of the order of 2.0, 2.35, and 5.0, respectively. Little can be done about the angular field of solid spacer filters (see “Fabry-Perot Filters with Solid Spacers” earlier in this section).

For metal-dielectric FP and for induced-transmission filters, effective indexes \( \mu^* \) of up to 3.2 and 2.0 have been reported.315

Wilmot and Schineller322 and Schineller and Flam323 have announced a filter consisting of a thin, plane-parallel fiber-optic face plate coated on both sides with all-dielectric mirrors. Since in such a filter the half-width is determined only by the thickness of the
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42.92

plate and the reflectivity of the coatings, and since the field of view depends on the ratio of the wavelength to the fiber diameter, the two quantities are independent. The measured transmittance of a 15-Å-half-width, 6-mm-diameter filter composed of 1.5-μm-diameter fibers was 30 percent and the shift in wavelength with angle of incidence was one-eighth that of a conventional filter.

Stability and Temperature Dependence of Bandpass Filters

The stability of the position of the transmission peak has been studied by many workers.259,276,324–332 The observed changes (up to 1 percent of λ) seem to depend greatly on the materials and manufacturing conditions. In filters with evaporated spacers both irreversible changes, probably due to changes in the structure of the films, and reversible changes due to the adsorption of water vapor, have been observed. Many manufacturers are now able to minimize these effects through the use of more stable materials, improved high-energy deposition methods (see Sec. 42.4) or accelerated artificial ageing processes. No changes were observed in solid spacer filters (see “Fabry–Perot Filters with Solid Spacers”).

Changes in the operating temperature normally do not significantly affect the half-widths and peak transmittances of medium- and wide-bandpass interference filters (see, for example, Refs. 333–336). An exception are filters that contain semiconductors that start to absorb significantly on heating (e.g., germanium) or on cooling (PbTe).337,338 The position of the transmission peaks shift linearly towards longer wavelengths with an increase in temperature, the magnitude of the shift depending largely on the spacer material. The temperature coefficient, expressed as a percentage change in λ per degree Celsius change in temperature, lies between $2 \times 10^{-4}$ and $3 \times 10^{-3}$ for filters with evaporated spacers for the 0.3- to 1.0-μm spectral region and between $2 \times 10^{-3}$ and $2 \times 10^{-2}$ for the infrared spectral region.259,345 It is of the order of $1 \times 10^{-3}$ for filters with mica swollen and quartz spacers. Unless temperature control is provided, under adverse conditions all of these temperature coefficients could lead to serious shifts of the transmission peaks of very narrow band filters (see earlier in this section). The temperature control can take the form of an external constant-temperature enclosure, or it might be built right into the filter. Eather and Reasoner276 and Mark et al.341 describe arrangements of the latter type in which two sensors embedded in the filters are used to control the current flowing through two transparent conducting coatings that surround the filter.

Deliberate changes in the temperature can be used for a fine-tuning of the transmission wavelength without having an adverse effect on the angular field of the filter.

Bandpass Filters for the XUV and X-ray Regions

The construction of good bandpass filters for the extreme ultraviolet is hampered by the lack of coating materials with suitable optical constants. However, certain metals in thin-film form can be used as rudimentary bandpass filters in the extreme ultraviolet. The primary process in these filters is absorption, although at times interference within the film may have to be considered to explain the spectral transmission characteristics fully.

The measured spectral transmittance of some of these materials is shown in Figs. 109 and 110. By increasing the thicknesses of the films, higher rejection ratios could be obtained at the expense of peak transmissions, and vice versa. The transmittance of the most promising material, aluminum, would be higher if it were not for the formation of absorbing oxide layers.

Many of the layers are self-supporting (Fig. 109). Others must be deposited onto a suitable transparent substrate (Fig. 110). Thin aluminum films are sometimes used for this
FIGURE 109 Measured extreme-ultraviolet and soft x-ray transmittance of several self-supporting metal films of indicated thicknesses.
purpose. Other materials used in the past are Zapon (cellulose acetate); collodion, Parlodion, and Celluloid (cellulose nitrates); Mylar (polyethylene teraphthalate); and Formvar (polyvinyl formal) (Figs. 109n,o). Any residual absorption in the substrate contributes, of course, to the overall-transmission curve. The preparation of self-supporting thin films is described by Novikov and by Sorokin and Blank. Because of their fragility, such films are usually mounted on a very fine mesh screen.

Multilayer Fabry-Perot interference filters for the soft x-ray region have also been constructed. However, thus far the only spectral measurements reported are nonnormal incidence reflection. The finesse of the filters is low and the modulation of the reflectance curve depends on the thickness of the spacer. The devices are useful for measurement purposes.

### 42.13 MULTILAYERS FOR TWO OR THREE SPECTRAL REGIONS

Increasingly, there are applications, especially in laser science, in which the spectral transmission and/or reflection has to be controlled at two or more wavelengths. The design and construction of such coatings is more difficult than that of systems for one wavelength region only, especially when the ratio of the wavelengths of interest is very large.

**Multilayers for Two Spectral Regions**

Costich was the first to specify the construction parameters of coatings having all possible combinations of low- and high-reflection behavior at wavelength ratios of 1.5-, 2.0-, and 3.0:1.0. His solutions were based on systems composed of quarter-wave layers or layers with other simple thickness relationships. Experimental results are in good agreement
with the calculated values. The calculated performance of two coatings not shown before are given in Fig. 111. Figure 112 presents the performance of a number of commercially produced coatings of this type. Systems for other wavelength ratios and for combinations of other reflection values are possible.

Sometimes coatings are required in which the reflectance is controlled for wavelength ratios of 10:1 or more. A systematic method for the design of such multilayers with different reflectance characteristics at the two wavelengths has been described. The calculated performance of such coatings for the wavelengths of 0.6328 and 10.6 μm are shown in Fig. 113.

Multilayers for Three Spectral Regions

For some laser applications, the reflectance or transmittance has to be controlled at three or more wavelengths. Solutions to such problems can also be found. Costich has given designs for all possible combinations of low and high reflection for the important special case of a set of wavenumbers $\sigma_1$, $2\sigma_1$, and $3\sigma_1$. The designs and performances of his solutions to this problem are shown in Fig. 114.

In principle, the method for the design of coating for two widely separated spectral regions mentioned here can be extended to the design of coatings for three or more
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FIGURE 113 Calculated performance of three multilayer coatings (a), (b), and (c) designed for two widely separated spectral regions. Columns 1 and 2 represent the performance of the multilayers in the visible and in the infrared spectral regions, respectively. The experimental measurements for one coating are also shown. (After Li.)

wavelengths. However, the number of layers required increases dramatically as the number of layers required for the longest wavelength region increases. Figure 115 shows the calculated performance of a coating that behaves like a high-reflection coating, a beam splitter, and an antireflection coating at 0.63, 2.52, and 10.6 μm, respectively.

42.14 PHASE COATING

In some applications, in addition to transmittance or reflectance requirements, special phase relationships have to be satisfied. These may be specific phase changes on reflection \( R \) or transmission \( T \) [Eqs. (23), (24)] for radiation incident at 0°. At other times it is required to displace or to deflect a beam without affecting its state of polarization. However, most frequently it is necessary to introduce a certain phase difference \( \Delta \) between \( \eta \)- and \( \xi \)-polarized light. Quarter-wave plates made of birefringent crystals are normally used to provide this phase difference.

Solutions to these and similar problems based on optical interference coatings can also be found. Porous films with an inclined columnar structure, formed in physical vapor deposition processes when the vapor is incident onto the substrate at an oblique angle, can also be birefringent. Such films have been proposed for the construction of phase retardation plates for use with normal incidence of the radiation. However, more frequently, solutions are based on the difference between the effective indices \( \eta \), \( \xi \) of thin films for obliquely incident radiation [Eq. (19)]. Azzam has shown that, when
FIGURE 114  Calculated performance of multilayer coatings on glass with various combinations of high and low reflectance at relative wave numbers ($\lambda_o/\lambda$) 1.0, 2.0, and 3.0. In the designs $H, L$ correspond to quarter-wave layers at $\lambda_o = 1.0 \mu m$. $n_H$, $n_L$, and $n_n$ were assumed to be 1.0, 1.52, 1.95, and 1.43, except in (a) where $n_H$, $n_L$ were 1.64, 1.38, respectively. (After Costich.*)

FIGURE 115  Calculated performance of a multilayer with different properties in three spectral regions. (After Li.*)
Phase-retarding reflectors are commonly designed for use at 45°. Many layers are required when the radiation is incident from the air side. The performances of two multilayers of this type with different phase differences are shown in Fig. 116. The multilayers are optimized to maintain a constant phase difference in the vicinity of the design wavelength. Coatings with other phase differences and reflectances can also be constructed. For example, an antireflection coating for 45° incidence in which the phase change is 180° is shown in Fig. 117.

When the radiation is incident on the layers from the substrate side, total internal reflection takes place. The design of thin-film phase retarders based on this approach has also been examined by Apfel and Azzam. Total-internal-reflection phase retarders operate over broader spectral regions (Fig. 118) but their size is limited by the weight and homogeneity of the prism materials.

More complex phase-retardation devices have been constructed in which the radiation is allowed to undergo two, three, or even four internal reflections. The performance of some typical total internal reflection devices that are based on the configurations of Figs. 65, 68, and 70 is shown in Fig. 119. For high-power laser beam delivery systems, front surface reflectors are usually employed (Fig. 120).

In Fig. 121 are shown the phase changes on reflection of a set of four metal/dielectric interferometer mirrors, all with $R > 0.97$, for which the differences in the phase changes on reflection for adjacent members in the set were approximately 90° over an extended spectral region. Other requirements for phase changes or phase-change differences can also be satisfied with thin films.

### 42.15 INTERFERENCE FILTERS WITH LOW REFLECTION

#### Reducing Reflection with a Thin Metal Film

A suitable thin metal film deposited onto a glass surface can act as a very efficient achromatic antireflection coating for light incident from the glass side (Fig. 122). The reflectance for light incident from the air side is not reduced and the transmittance suffers as a result of the absorption within the film. By combining such films with additional layers, attractive colored sunglasses or architectural coatings are obtained.
FIGURE 118 Phase retarders based on total internal reflection, consisting of three layers on glass and operating at an angle of incidence of 45°: (a) 0° phase retardation (after Cojocaru et al.); (b) 90° phase retardation (after Spiller et al.).

FIGURE 119 90° phase retardation devices based on (a) 2, (b) 3, and (c) 4 total internal reflections (after King, Clapham, and Filinski). The angle of incidence on the first reflecting surface is indicated in the diagrams.

FIGURE 120 Phase retardation devices for λ = 10.6 μm based on multiple reflections from surfaces coated with opaque silver films and single layers of ZnS having specified thicknesses. (a), (b) 0° phase retardation device for all angles of incidence (after Azzam et al.); (c), (d) 90°, 180° phase retardations (after Thonn et al.).
Black Absorbers

Black absorbers efficiently absorb the radiation incident upon them in a specified spectral region. They are used, for example, to control radiant energy,\textsuperscript{364} to remove stray light in optical systems, to enhance contrast in display devices,\textsuperscript{365} and to increase the signal-to-noise ratio in multiplexers.\textsuperscript{366} Black absorber coatings are based on interference in thin films and generally consist of an opaque metal layer and one or more dielectric layers interspersed with partially transparent metal layers (Figs. 20, 63). They can be designed for first- and second-surface application (Fig. 123). Coatings of this type can also be designed for the ultraviolet and infrared spectral regions.\textsuperscript{367}

Neutral Attenuators

Conventional metallic film attenuators described under “Neutral Filters” cannot readily be placed in series because multiple reflections between the components may result in unpredictable density values. However, by using metal and dielectric layer combinations of appropriate optical constants and thicknesses, it is possible to reduce the reflection of the

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure121.png}
\caption{Normal incidence phase changes on reflection of a set of four highly reflecting mirrors for Michelson interferometers. (After Piotrowski et al.\textsuperscript{499})}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure122.png}
\caption{Spectral reflectance of thin chromium films on glass for light incident from the substrate side. The transmittance of the layers at $\lambda = 0.565$ $\mu$m is indicated. (After Pohlack.\textsuperscript{363})}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure123.png}
\caption{Calculated performance of a five-layer metal/dielectric black absorber. (After Dobrowolski.\textsuperscript{500})}
\end{figure}
metallic film from one or both sides of the substrate. The experimental results for one such attenuator are given in Fig. 124b.

**Other Interference Filters**

It is possible, using a similar approach, to reduce the reflection of narrow-bandpass filters, cut-off filters, and other filter types. In particular, low-reflection narrowband interference filters for welding applications have been described by Jacobsson. The experimental performance of a bandpass filter and of a long-wavelength cut-off filter are given in Figs. 124c and d. In both cases, the luminous reflectance has been reduced by an order of magnitude over that of a conventional design. However, this is at the expense of the transmittance.

### 42.16 REFLECTION FILTERS AND COATINGS

**Metallic Reflectors**

The Fresnel reflection coefficient of an interface between two semi-infinite media of complex refractive indices \( \tilde{n}_m, \tilde{n}_s \), for polarized radiation incident at nonnormal angle is given by

\[
R = \left| \frac{\eta_r - \eta_i}{\eta_r + \eta_i} \right|^{22}
\]

(79)
where $\eta_s, \eta_m$ are given by Eq. (19). When $\bar{n}_m, \bar{n}_s$ correspond to air and the metal, respectively, and when the angle of incidence is zero, the preceding expression reduces to:

$$R = \frac{(n_s - 1)^2 + k_s^2}{(n_s + 1)^2 + k_s^2}$$

(80)

If the substrate is opaque, this represents the total energy reflected, the remaining energy being absorbed within the material.

Metal reflectors are most commonly made by vacuum deposition of the material onto a suitable glass or quartz substrate. Before deposition, aluminium or beryllium mirror surfaces are sometimes first chemically plated with a nickel-phosphorus alloy (Kanigen process). Such deposits have excellent adhesion to the substrate and have a very hard surface that can be optically polished before coating.375

Visible, infrared, and ultraviolet spectral-reflectances of some of the more commonly used metals are shown in Figs. 125 and 126. Using Eq. (80) and the optical constants in Palik’s handbook,48,49 the spectral reflectances for many additional metals can be

FIGURE 125 Reflectances of some metals. (After Drummeter and Hass.109)

FIGURE 126 Visible and infrared reflectance of certain metals. (Al, after Bennett et al.;304 Ag and Au, after Bennett and Ashley;305 Cu, after Hass and Hadley;306 Rh and Pt, after Hass and Fowler, see Drummeter and Hass.)109
calculated. Silver has the highest visible and infrared reflectance, and hence is used for interferometer mirrors and interference filters. Exposed silver films tarnish readily.

Aluminum has the broadest high-reflectance region of all metals and is commonly used in front-surface mirrors. It would reflect highly down to 0.1 \( \mu \text{m} \) were it not for the absorption below 0.18 \( \mu \text{m} \) of the thin oxide layer that starts to form seconds after deposition.\(^{376,377}\)

Some of the highest known reflectances in the ultraviolet are shown in Fig. 127. At still shorter wavelengths, all materials have refractive indices that are close to unity and extinction coefficients that are rather small. It follows from Eq. (80) that normal-incidence reflectances in that part of the spectrum are small. However, for angles of incidence greater than the critical angle \( \theta_c \),

\[
\theta_c = \cos^{-1} \left[ \sqrt{\frac{2}{n_s^2}} \right]
\]

(81)
total external reflection occurs resulting in high reflectances. Measured values of oblique angle reflection coefficients in the 0.0023- to 0.019-\( \mu \text{m} \) spectral region for a number of materials are given by Lukirkii et al.\(^{376,379}\). Typical spectral reflectance curves are shown in Fig. 128. An Al reflectance of 0.987 for an angle of incidence of 80° at 0.0584 \( \mu \text{m} \) has been reported by Newnam.\(^{380}\) Such coatings can be used in near-grazing incidence optics.

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**FIGURE 127** Measured ultraviolet reflectance of certain materials. (Pt, Au, and ZnS, after Hunter;\(^{507}\) Ir, after Hass et al.;\(^{508}\) Rh, after Cox et al.;\(^{509}\) Re, W, after Cox et al.;\(^{509}\) Os, after Cox et al.;\(^{509}\) and SiC, after Seely.\(^{511}\))

**FIGURE 128** Measured x-ray reflectances of (a) Ni, Au, and Al films at 85° (after Malina and Cash\(^{512}\)); and (b) Al, Si, C, and CVD SiC films at 89° angle of incidence (after Windt et al., 1988\(^{513}\)).
OPTICAL PROPERTIES OF FILMS AND COATINGS

FIGURE 129 Measured difference between the reflectances of protected and unprotected aluminum mirrors. Curve 1: coating 4 of Fig. 130b; curves 2 and 3: 0.1122 ± 0.002- and 0.0752 ± 0.001-μm-thick films of MgF₂ and SiO₂, respectively, on aluminum. (After Bennett.514)

Metal-dielectric Reflectors

Protective Coatings. For many applications, the thin aluminum oxide layer on an aluminum surface does not offer sufficient protection against abrasion and chemical attack, and therefore aluminum mirrors are often overcoated with single SiO₂ or MgF₂ protective layers. Such mirrors can be repeatedly cleaned with water and even withstand boiling in salt water. The deterioration of the ultraviolet reflectance of aluminum mirrors due to oxidation can be partially avoided by covering the freshly deposited aluminum layer immediately with a suitable coating of MgF₂ or LiF. The reflectances of such overcoated aluminum reflectors are shown in Fig. 131. Their variation with angle of incidence in the 0.03- to 0.16-μm spectral region is discussed by Hunter.387

The reflectance of unprotected and protected silver mirrors has been investigated by Burge et al. Highly adherent and chemically stable mirrors with a reflectance in excess of 0.95 for wavelengths greater than 0.5 μm have been reported.

The reflectance of protected metal mirrors at oblique angles of incidence in the infrared part of the spectrum can be seriously reduced at the short-wavelength side of the Reststrahlen peak of the material used for its protection.

Enhancement of Reflection. By depositing a quarter-wave stack (see "Nonabsorbing

FIGURE 130 Reflectance of very durable overcoated metal mirrors: (a) silver mirrors—curve 1, protected front-surface mirror (after Denton); curve 2, enhanced reflection (after Vvedenskii); curve 3, extended reflection (after Song et al.); (b) aluminum mirrors—curve 4, with four layers of MgF₂ and CeO₂ (after Hass); curve 5, with four layers of SiO₂ and TiO₂ (after AIRCO); curve 6, with four layers MgF₂ and ZnS (after Furman and Stolo).
FIGURE 131 Measured spectral-reflectance curves of unprotected aluminum and aluminum overcoated with MgF$_2$ and LiF films of indicated thicknesses. (Al + LiF coating, after Cox et al., all other curves after Canfield et al.)

[AB]$^n$ and [AB]$^n$A Multilayers—Theory” and “Periodic Multilayers of the [(0.5A)B(0.5A)]$^n$ Type” in Sec. 42.7) onto the metal mirror, its reflectance can be enhanced considerably. The thickness of the first layer should be adjusted to compensate for the phase change on reflection at the metal surface. The spectral-reflectance curve dips on either side of the high-reflection region whose width is governed by the considerations under “Width of the High-Reflectance Zone” in Sec. 42.7 and which can be somewhat enhanced by the use of a half-wave outermost layer (Fig. 130). The measured spectral characteristics of three metal-dielectric reflectors for the ultraviolet region are shown in Fig. 132.

The reflectance of silver, although very high in the visible, falls off rapidly in the near-ultraviolet. Attempts to enhance the reflectance in that part of the spectrum and, at the same time, to protect the silver from tarnish, have been successful (Fig. 130).

A different kind of reflection enhancement has been reported for the extreme ultraviolet. By depositing semitransparent platinum films onto different substrates, the opaque-film reflectances of 19.3 and 12.8 percent at 0.0584 and 0.0736 μm were increased by up to 2.8 and 3.8 percent, respectively. For space applications, suitably thick aluminum films on iridium are expected to yield reflectances as high as 40 and 52 percent at the same wavelengths (Fig. 133). Other proposed reflectance-increasing combinations can be found in Madden et al.

FIGURE 132 Enhanced ultraviolet reflectances of semitransparent aluminum films obtained through the addition of quarter-wave stacks. Curves 1 and 2: 11 layers of PbF and MgF$_2$ (after Les et al.); curve 3: nine layers of Sb$_2$O$_5$ and MgF$_2$ (after Les and Les).

FIGURE 133 Calculated spectral reflectance curves of iridium overcoated with different thicknesses of unoxidized films of aluminum. (After Hass and Hunter.)
Selectivity Metal-Dielectric Reflectors. Several types of coatings that reflect highly in one spectral region, but not in another, have been developed in the past for different applications. Hadley and Dennison presented the theory and experimental results of reflection interference filters for the isolation of narrow spectral regions (Fig. 134a). Very narrow reflection filters have been described by Zheng (Fig. 134b). High-infrared and low-visible reflectance coatings (Fig. 63) are used to control the temperature of satellites. These coatings could also be used to remove visible light from infrared optical systems. Several reflectors designed to reduce stray visible light in ultraviolet systems are shown in Fig. 135.

Multiple-Reflection Filters

Metal and Metal-dielectric Multiple-reflection Filters. Metals such as silver, copper, gold, and metal-dielectric coatings of the type shown in Fig. 63 used in a multiple-reflection arrangement should make cut-off filters with excellent rejection, sharp transition, and a long, unattenuated pass region far superior to those available with transmission filters.

Multiple-reflection Filters Made of Thin-film Interference Coatings. Interference coatings for use in a multiple-reflection filter need not be deposited onto substrates that
transmit well in the spectral region of interest, but they should be used at small angles of incidence if disturbing effects due to polarization are not to occur. The following are examples of some of the difficult filtering problems that can be easily solved with multiple-reflection filters composed of interference coatings, providing that there is space to use a multiple-reflection arrangement.

It is difficult to provide adequate blocking with transmission filters for narrow-bandpass filters of the type shown in Figs. 93 to 98 without considerably reducing the peak transmittance. This is done readily with a multiple-reflection filter composed of quarter-wave stacks of the same materials used for the construction of the bandpass filter and centered at the same wavelength (Fig. 136).

It should be possible to construct highly efficient short-pass filters with a very long and low rejection by using broadband reflectors consisting of several contiguous stacks (see “All-Dielectric Broadband Reflectors” in Sec. 42.8).

The use of a narrowband transmission filter in a multiple-reflection arrangement of the type shown in Fig. 3 results in a high-attenuation narrowband rejection filter surrounded by regions of high transmission (Fig. 137). However, such devices must be used with well-collimated light.

The transmittance curves of three multiple-reflection bandpass filters are shown in Figs. 138a–c. By using a number of multiple-reflection filters with sharp features, it is possible to separate signals transmitted by radiation of different, closely spaced wavelengths with a very low crosstalk and small insertion loss.

Additional information on reflection coatings and filters will be found in the reviews by Hass et al. and by Lynch.

42.17 SPECIAL-PURPOSE COATINGS

Space considerations limit the detailed description of coatings and filters for specific applications. Absorbing multilayer coatings on glass for enhancing the visual appearance, thermal and illumination control, and as “one-way mirrors” find applications in architecture and in the automotive industry. They are also used in solar-energy conversion and have been proposed for radiative cooling. Thin-film coatings are used in optical recording media and in optical multiplexers/demultiplexers. Bistable Fabry-Perot structures are
FIGURE 138 Measured spectral characteristics of broad- and narrowband multiple-reflection filters for the infrared and ultraviolet spectral regions: (a) eight reflections (solid curves) from identical quarter-wave stacks (dotted); (b) six reflections (solid) from each of two quarter-wave stacks (dotted, dashed) tuned to different wavelengths (after Valeev); (c) commercial multiple-reflection interference filter (Schott and Gen.).

proposed for use as light switches in optical computers. Special filters and coatings are used in colorimetry, radiometry, detectors, and in high-contrast display devices. Consumer-oriented products include various kinds of decorative coatings, as well as coatings for the protection of documents and products from counterfeiting. There is no doubt that in the future even more applications will appear for optical multilayer coatings. Some of these will require very complex spectral characteristics. Methods for the design of such coatings have existed for some time now. At the time of writing, feasibility of the construction of such filters in the laboratory has also been demonstrated (Fig. 139).

FIGURE 139 Calculated and experimentally measured reflectance of a filter that approximates the silhouette of the Taj Mahal. (After Sullivan and Dobrowolski.)
42.18 ACKNOWLEDGMENTS

The author would like to acknowledge the help and encouragement of his colleagues, Brian T. Sullivan, Li Li, Claude Montcalm, Pierre Verly, Jeffrey Wong, and Allan Waldorf. He is also grateful to Philip Baumeister, Art Guenther, and Gaétan Duplain for their comments on the manuscript.

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