# CHAPTER 14
LIQUID CRYSTALS

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## 14.1 GLOSSARY

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$C_i$</td>
<td>reduced elastic constants</td>
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<tr>
<td>$d$</td>
<td>liquid crystal thickness</td>
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<td>$E$</td>
<td>electric field</td>
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<tr>
<td>$E_a$</td>
<td>activation energy</td>
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<td>$e_c$</td>
<td>electroclinic coefficient</td>
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<td>$F$</td>
<td>Onsager reaction field</td>
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<td>$f_i$</td>
<td>oscillator strength</td>
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<td>$f_c$</td>
<td>crossover frequency</td>
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<td>$h$</td>
<td>cavity field factor</td>
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<td>$K_a$</td>
<td>elastic constants</td>
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<td>$k$</td>
<td>Boltzmann constant</td>
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<td>$l$</td>
<td>aspect ratio</td>
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<tr>
<td>$n_{e,o}$</td>
<td>refractive indices</td>
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<tr>
<td>$N$</td>
<td>molecular packing density</td>
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<tr>
<td>$P_4$</td>
<td>order parameter of the fourth rank</td>
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<tr>
<td>$P_s$</td>
<td>spontaneous polarization</td>
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<tr>
<td>$S$</td>
<td>order parameter of the second rank</td>
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<td>$T$</td>
<td>temperature</td>
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<td>$T_c$</td>
<td>clearing temperature</td>
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<td>$V_n$</td>
<td>mole volume</td>
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<td>$V$</td>
<td>voltage</td>
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<tr>
<td>$V_b$</td>
<td>bias voltage</td>
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<tr>
<td>$V_{th}$</td>
<td>threshold voltage</td>
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<tr>
<td>$Z$</td>
<td>number of electrons</td>
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<tr>
<td>$\alpha_{ij}$</td>
<td>molecular polarizability</td>
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14.2 INTRODUCTION

Liquid crystals possess physical properties that are intermediate between conventional fluids and solids. They are fluidlike, yet the arrangement of molecules within them exhibits structural order. Three types of liquid crystals have been studied extensively: (1) thermotropic, (2) lyotropic, and (3) polymeric. Among these three, the thermotropic liquid crystals have been studied extensively and their applications have reached mature stage. The potential applications are in areas such as flat panel displays, light switches, temperature sensors, etc. Lyotropic liquid crystals are receiving increasing scientific and technological attention because of the way they reflect the unique properties of their constituent molecules. Polymeric liquid crystals are potential candidates for electronic devices and ultra-high-strength materials. Thermotropic liquid crystals can exist in three phases: (1) nematic, (2) cholesteric, and (3) smectic. Their structures and representative molecules are illustrated in Fig. 1.

14.3 PHYSICAL PROPERTIES OF THERMOTROPIC LIQUID CRYSTALS

Optical Properties

Refractive indices and absorption are fundamentally and practically important parameters of an LC compound or mixture. Almost all the light modulation mechanisms are involved with refractive index change. The absorption has a crucial impact on the photostability or lifetime of the liquid crystal devices. Both refractive indices and absorption in the visible spectral region are determined by the electronic structures of the liquid crystal studied.
Electronic Structures. Three types of electronic transitions are often encountered in a liquid crystal compound. They are: (1) $\sigma \rightarrow \sigma^*$ (excited states of $\sigma$-electrons), (2) $n \rightarrow \pi^*$ (excited states of $\pi$-electrons), and (3) $\pi \rightarrow \pi^*$ transitions.

$\sigma \rightarrow \sigma^*$ Transitions. The majority of liquid crystals discovered so far consist of either saturated cyclohexane rings, unsaturated phenyl rings, or a combination of both. In addition to affecting the mesogenic range, these rings also make the primary contribution to absorption and refractive indices of the liquid crystal molecule. For a cyclohexane ring, only $\sigma$-electrons are present. The $\sigma \rightarrow \sigma^*$ transitions take place at vacuum ultraviolet region ($\lambda < 180$ nm). Thus, the absorption of a totally saturated LC compound or mixture becomes negligible in the visible spectral region.

$n \rightarrow \pi^*$ Transitions. In molecules containing a heteroatom (such as oxygen or nitrogen), the highest filled orbitals in the ground state are generally the nonbonding, essentially atomic $n$ orbitals, and the lowest unfilled orbitals are $\pi^*$. Thus, the $n \rightarrow \pi^*$ transitions are the excitation mechanism for these special molecules. The oscillator strength of the $n \rightarrow \pi^*$ transition is not very strong. Thus, its contribution to refractive indices is not significant.

$\pi \rightarrow \pi^*$ Transitions. The $\pi \rightarrow \pi^*$ electronic transitions of a benzene molecule have been studied extensively using the molecular-orbital theories. These results can be used to explain the UV absorption of compounds containing a phenyl ring. From the Group Theory, a benzene molecule belongs to the $D_{6h}$ point group, but a phenyl ring belongs to the $C_{2v}$ group. Obviously, the degree of symmetry of a phenyl ring in a liquid crystal compound is lower than an unbounded benzene molecule due to the constraints from the...
FIGURE 2. A simplified energy diagram of $\pi \rightarrow \pi^*$ electronic transitions of a benzene molecule. The transition intensity for the $\lambda_1$-band is very strong, $\lambda_2$-band is strong, and $\lambda_3$-band is weak. The $\lambda_1$-band has a twofold degeneracy.

FIGURE 3. Polarized absorption spectrum of 5CB. $\parallel$ and $\perp$ represent extraordinary and ordinary rays and the middle curve is for the unpolarized light. Sample: 1 percent 5CB dissolved in ZLI-2359. Cell thickness = 6 $\mu$m. $T_r = 0.865.$

Connecting segments. If a liquid crystal molecule contains a phenyl ring, its absorption spectrum would be similar to that of a pure benzene molecule. The degree of similarity depends on how many conjugated bonds are attached to the phenyl ring.

The simplified $\pi \rightarrow \pi^*$ electronic transition diagram of a benzene molecule is shown in Fig. 2. From selection rules, the $^1A_{1g} \rightarrow ^1E_{1u}$ is a spin- and symmetry-allowed transition. In a pure benzene molecule, this band has a twofold degeneracy and large extinction coefficient. But in a phenyl ring, this degeneracy gradually splits as the conjugation length increases. The $^1A_{1g} \rightarrow ^1B_{1u}$ is a spin-allowed, symmetry-forbidden, but vibronically allowed transition. Its extinction coefficient is modest. Finally, the $^1A_{1g} \rightarrow ^1B_{2u}$ is a spin-allowed but symmetry-forbidden transition. Its extinction coefficient is very weak. In the three-band model for understanding the refractive index dispersion, only the two major $\pi \rightarrow \pi^*$ transitions, the $^1A_{1g} \rightarrow ^1E_{1u}$ (designated as $\lambda_1$-band) and the $^1A_{1g} \rightarrow ^1B_{1u}$ ($\lambda_2$-band), and the $\sigma \rightarrow \sigma^*$ transitions ($\lambda_0$-band) are included. The polarized absorption spectra of 5CB is shown in Fig. 3. Indeed, the $\lambda_1$-band consists of two closely separated bands; the averaged wavelength is $\lambda_1 = 210$ nm. The $\lambda_2$ is centered at 282 nm, and its dichroic ratio is large. As temperature increases, dichroic ratio decreases gradually.

Refractive Index Dispersions. Taking all three bands into consideration, the following expressions for refractive indices have been derived:

$$n_e \approx 1 + g_{\omega_0} \frac{\lambda^2 \lambda_0^2}{\lambda^2 - \lambda_0^2} + g_{\omega_1} \frac{\lambda^2 \lambda_1^2}{\lambda^2 - \lambda_1^2} + g_{\omega_2} \frac{\lambda^2 \lambda_2^2}{\lambda^2 - \lambda_2^2}$$

$$n_o \approx 1 + g_{\omega_0} \frac{\lambda^2 \lambda_0^2}{\lambda^2 - \lambda_0^2} + g_{\omega_1} \frac{\lambda^2 \lambda_1^2}{\lambda^2 - \lambda_1^2} + g_{\omega_2} \frac{\lambda^2 \lambda_2^2}{\lambda^2 - \lambda_2^2}$$

where $g_i \sim NZ_i f_i$ are proportionality constants; $Z_0$ is the number of responsible $\sigma$ electrons and $Z_1 = Z_2$ is the number of $\pi$ electrons in an LC compound. These $g_i$’s determine the
temperature effect of the refractive indices. In the off-resonant region, Eq. (1) can be expanded into the Cauchy-type formula:

\[ n_{e,o} = A_{e,o} + \frac{B_{e,o}}{\lambda^2} + \frac{C_{e,o}}{\lambda^4} + \ldots \] (2)

where \( A_{e,o}, B_{e,o}, \) and \( C_{e,o} \) are expansion coefficients. For simplicity, we may just keep the first two terms for fitting the experimental data.

**Molecular Vibrational Absorptions.** In the visible spectral region, directors’ fluctuation-induced light scattering surpasses the absorption loss. Thus, absorption has to be measured at the isotropic state. Figure 4 shows the absorption of 5CB and MBBA at \( T \sim 60^\circ C \). At \( \lambda \gg \lambda_c \), absorption decreases as \( \lambda \) increases. In the near IR region, some harmonics of molecular vibrational bands appear, resulting in enhanced absorption. As

![Absorption Coefficient](image_url)

**FIGURE 4** Absorption coefficient of (a) 5CB and (b) MBBA at \( T \sim 60^\circ C \) in the visible and near IR region. The clearing point of 5CB and MBBA is 35.2 and 46°C, respectively. Light scattering is neglected under this isotropic phase.
wavelength extends to IR, molecular vibrational absorptions start at around 3.3 μm.\textsuperscript{12} Figure 5 shows the normalized transmission of a ~15-μm-thick, parallel-aligned E-7 LC mixture using an unpolarized light. The truncated transmission at λ = 40 μm is due to the KRS-5 window, and not due to the liquid crystal. Although there exist some localized vibrational bands, there are broad regions where the transmission is reasonably high and useful electro-optic effect can be realized.

**Dielectric Constants**

Dielectric constants and their anisotropy affect the sharpness of the voltage-dependent optical transmission curve of an LV device and its threshold voltage. Maier and Meier\textsuperscript{13} have developed a theory to correlate the microscopic molecular parameters with the macroscopic dielectric constants of anisotropic LCs:

\[
\varepsilon_\parallel = N h F ((\alpha_\parallel + (F \mu^2 / 3kT) [1 - (1 - 3 \cos^2 \beta) S])
\]

\[
\varepsilon_\perp = N h F ((\alpha_\perp + (F \mu^2 / 3kT) [1 + (1/2)(1 - 3 \cos^2 \beta) S])
\]

\[
\Delta \varepsilon = N h F ((\alpha_\parallel - \alpha_\perp) - (F \mu^2 / 2kT)(1 - 3 \cos^2 \beta) S)
\]

where \( N \) is the molecular packing density, \( h = 3 \varepsilon/(2\varepsilon + 1) \) is the cavity field factor, \( \varepsilon = (\varepsilon_\parallel + 2 \varepsilon_\perp)/3 \) is the averaged dielectric constant, \( F \) is the Onsager reaction field, \( \alpha_\parallel \) and \( \alpha_\perp \) are the principal elements of the molecular polarizability tensor, \( \beta \) is the angle between the dipole moment \( \mu \) and the principal molecular axis, and \( S \) is the order parameter of the second rank. From Eq. (4), the dielectric constants of anisotropic liquid crystals are influenced by the molecular structure, temperature, and frequency. These individual effects are discussed separately.

**Structural Effect.** For a nonpolar liquid crystal compound, its dipole moment \( \mu = 0 \). Thus, its \( \Delta \varepsilon \) is expected to be small and its magnitude is proportional to the differential

![FIGURE 5 IR transmission of a 15-μm, parallel-aligned E-7 LC mixture. The truncation at the ~40-μm region is due to the KRS-5 substrates used.](image-url)
molecular polarizability, similar to birefringence. On the other hand, for an LC molecule containing a polar group, such as the cyano, isocyanate, fluoro, or chloro group, its $\Delta \varepsilon$ can be positive or negative depending on the position(s) of the polar group(s). If $\beta \approx 0$, i.e., the dipole moment of the polar group is along the principal molecular axis, $\Delta \varepsilon$ is large and positive. Cyano-biphenyls are such examples. The $\Delta \varepsilon$ of 5CB is about 10 at $T = 20^\circ$C and $f = 1$ kHz. These positive $\Delta \varepsilon$ materials are useful for parallel or twist alignment. On the contrary, if $\beta > 55^\circ$, $1 - 3 \cos^2 \beta > 0$ and $\Delta \varepsilon$ may become negative depending on the dipole moment as indicated in Eq. (3c). The negative $\Delta \varepsilon$ materials are useful for perpendicular alignment.

**Temperature Effect.** In general, as temperature rises, $\varepsilon_i$ decreases, but $\varepsilon_\perp$ increases gradually resulting in a decreasing $\Delta \varepsilon$. From Eq. (3c), the temperature dependence of $\Delta \varepsilon$ is proportional to $S$ for the nonpolar LCs and $S/T$ for the polar LCs. At $T > T_c$, the isotropic phase is reached and dielectric anisotropy vanishes.

**Frequency Effect.** In an aligned LC, the molecular rotation around the short axis is strongly hindered. Thus, the frequency dispersion occurs mainly at $\varepsilon_i$; $\varepsilon_\perp$ remains almost constant up to microwave region. Figure 6 shows the frequency-dependent dielectric constants of the M1 LC mixture (from Roche) at various temperatures. As the frequency increases, $\varepsilon_i$ decreases and beyond the crossover frequency $f_c$, $\Delta \varepsilon$ changes sign. The dielectric anisotropies of M1 are symmetric at low and high frequencies. The crossover frequency is sensitive to temperature. As temperature rises, the $\varepsilon_i$ and $\varepsilon_\perp$ of M1 both decrease slightly. However, the frequency-dependent $\varepsilon_i$ is strongly dependent on temperature, but $\varepsilon_\perp$ is inert. Thus, the crossover frequency increases exponentially with temperature; for example, $f_c \sim \exp (-E_a/kT)$; $E_a$ is the activation energy. For M1 mixture, $E_a = 0.96$ eV. Dual frequency effect is a useful technique for improving the response times of an LC device. In the dual frequency effect, a low frequency ($<f_c$ where $\Delta \varepsilon > 0$) electric field is used to drive the device to its on state, and during the decay period a high frequency ($>f_c$ where $\Delta \varepsilon < 0$) electric field is applied to speed up the relaxation time. From a material standpoint, an LC mixture with low $f_c$ and large $\Delta \varepsilon$ at both low and high frequencies is beneficial. But for a single LC substance (such as cyano-biphenyls), its $f_c$ is usually too high ($>10^9$ Hz) to be practically employed. In the microwave region, the LC absorption is small and birefringence relatively large. Thus, liquid crystals are useful electro-optic media in the spectral range covering from UV, visible, and IR to microwave. Of course, in each spectral region, an appropriate LC material has to be selected.

![FIGURE 6 Experimental results on frequency and temperature dependences of $\varepsilon_i$ and $\varepsilon_\perp$ of a Roche mixture M1. Note that the crossover frequency increases exponentially with temperature.](image.png)
Elastic Constants

Both threshold voltage and response time are related to the elastic constant of the LC used. Several molecular theories have been developed for correlating the Frank elastic constants with molecular constituents. Here we only introduce two theories: (1) the mean-field theory,\textsuperscript{22,23} and (2) the generalized van der Waals theory.\textsuperscript{24}

\textbf{Mean-field Theory.} In the mean-field theory, the three elastic constants are expressed as:

\begin{align*}
  K_i &= C_{ii} \cdot \gamma_i^{-7/3} \cdot S^2 \quad (4a) \\
  C_{ii} &= (3\gamma/2)(L \cdot m^{-1} \cdot \gamma_i^{-5/3})^{1/3} \quad (4b)
\end{align*}

where $C_{ii}$ is called the reduced elastic constant; $V_n$ is the mole volume; $L$ is the length of the molecule; $m$ is the number of molecules in a steric unit in order to reduce the steric hindrance; $\gamma_{11} = \gamma_{22} = 2\pi/L$ and $\gamma_{33} = (2\pi/L)^2$ where $x$, $y$, and $z$ are the averaged molecular distances in the $x$, $y$, and $z$ directions; and $A = 1.3 \times 10^{-6}$ erg cm$^{-6}$. The temperature-dependent elastic and reduced constants for a liquid crystal PAA (4,4'-dimethoxy-azoxybenzene) are shown in Fig. 7.

From Eq. (4), the ratio $K_{11} : K_{22} : K_{33}$ is equal to 1 : 1 : ($z/x$)$^2$ and the temperature dependence of elastic constants is basically proportional to $S^2$. This $S^2$ dependence has been experimentally observed for many LCs. However, the prediction for the relative magnitude of $K_i$ is correct only to the first order. Experimental results indicate that $K_{22}$ often has the lowest value, and the ratio of $K_{33}/K_{11}$ can be either greater or less than unity.

For PAA at 400 K,\textsuperscript{25} $K_{11} : K_{22} : K_{33} = 2 : 1 : 3.2$.

\textbf{Generalized van der Waals Theory.} Gelbart and Ben-Shaul\textsuperscript{24} extended the generalized
van der Waals theory for explaining the detailed relationship between elastic constants and molecular dimensions, polarizability, and temperature. They derived the following formula for nematic liquid crystals:

$$K_a = a_i(P_2) + b_i(P_4)$$  \hspace{1cm} (5)

where $a_i$ and $b_i$ represent sums of contributions of the energy and the entropy terms. They depend linearly on temperature; $P_2$ and $P_4$ are the order parameter of the second and the fourth rank, respectively. In general, the second term may not be negligible in comparison with the $S^2$ term depending on the value of $P_2$. Temperature-dependent $P_2$ and $P_4$ of 5CB are shown in Fig. 8. As temperature increases, both $S$ and $P_2$ decrease. The $P_2$ of 5CB changes sign at $T < T_c = 58^\circ C$. The ratio of $P_4/S$ is about 15 percent at $T = 20^\circ C$. If the $P_4$ of an LC is much smaller than $S$ in its nematic range, Eq. (5) is reduced to the mean-field theory, or $K_a = S^2$. The second term in Eq. (5) is responsible for the difference between $K_{11}$ and $K_{33}$.

Viscosities

Rotational viscosity ($\gamma_1$) of an aligned liquid crystal represents an internal friction among LC directors during the rotation process. The magnitude of $\gamma_1$ depends on the detailed molecular constituents, structure, intermolecular association, and temperature. As temperature increases, $\gamma_1$ decreases rapidly. Rotational viscosity is an important parameter for many electro-optical applications employing liquid crystals, because the response time of the LC device is linearly proportional to $\gamma_1$. Several theories, including rigorous and semiempirical, have been developed in an attempt to account for the origin of the LC viscosity. However, owing to the complicated anisotropic attractive and steric repulsive interactions among LC molecules, these theoretical results are not yet completely satisfactory. Some models fit certain LCs, but fail to fit others.
In the molecular theory developed by Osipov and Terentjev, all six Leslie viscosity coefficients are expressed in terms of microscopic parameters. From the Osipov-Terentjev theory, the parameters affecting the rotational viscosity of an LC are:

1. **Activation energy**: An LC molecule with low activation energy leads to a low viscosity.
2. **Moment of inertia**: An LC molecule with linear shape and low molecular weight would possess a small moment of inertia and exhibit a low viscosity.
3. **Intermolecular association**: An LC with weak intermolecular association (e.g., not form dimer) would reduce the viscosity significantly.
4. **Temperature**: Elevated temperature operation of an LC device may be the easiest way to lower viscosity. However, birefringence, elastic, and dielectric constants are all reduced as well.

### 14.4 Physical Mechanisms for Modulating Light

Several mechanisms for modulating light using liquid crystals have been developed. These include: (1) dynamic scattering, (2) the guest-host effect, (3) field-induced nematic-cholesteric phase change, (4) field-induced director axis reorientation, (5) the laser-addressed thermal effect, and (6) light scattering by micron-sized droplets.

#### Dynamic Scattering

When a dc or low-frequency ac field is applied to a nematic liquid crystal cell, the ionic motion due to the conductivity anisotropy induces electro-hydrodynamic flow which is coupled to the molecular alignment via viscous friction. The liquid crystal becomes turbulent and scatters light strongly. Usually a dc field of the order of $10^4$ V/cm is required to generate such effects. The contrast ratio of the maximum and minimum transmitted light intensities is about 20:1, and the response time is about 200 ms.

For pure liquid crystal compounds, the conductivity should be very low. The observed nonnegligible conductivity of liquid crystals may originate from the impurities or dopants. Because of the finite current flow involved, the dynamic scattering mode encounters problems such as large power consumption, instability, and short lifetime. The dc field also tends to trigger undesirable electrochemical reactions among the liquid crystal molecules, thus generating more impurities and degrading their chemical stability. The dynamic scattering mode does have the advantage that it does not require a polarizer in order to modulate the light. It has been used in watches and photoaddressed light valves in the early stage of liquid crystal device development, but has since been replaced by the field effect owing to the problems mentioned previously.

#### Guest-Host Effect

Guest-host systems are formed by dissolving a few percent (1 to 5 percent, limited by the solubility) of dichroic dye in the liquid crystal. The host material should be highly transparent in the spectral region of interest. The dichroic dye molecules should have a strong absorption for one polarization and weak in another in order to enhance the
contrast ratio. In the field-off state, the dye molecules are nearly parallel to the incident light polarization so that high absorption is obtained. When the liquid crystal directors are reoriented by the field, the dye molecules will also be reoriented, and the absorption is reduced.

Contrast ratio of the device employing the guest-host system is affected by the dichroic ratio and concentration of the dye molecules, and by the cell thickness. For a given cell thickness, a higher dye concentration leads to a higher contrast ratio, but the corresponding transmission is reduced and the response times are lengthened. Many dichroic dyes are available in the visible region. Due to the long conjugation of dye molecules, their viscosity is usually very large. The mixture containing merely 5 percent dyes is enough to result in a significantly larger viscosity. A typical contrast ratio of the guest-host system is 50 : 1. The guest-host effects in ferroelectric liquid crystals have also been investigated where a response time of less than 100 \( \mu \)s has been observed.33

Field-induced Nematic-Cholesteric Phase Change

Electric field-induced nematic-cholesteric phase change has been observed experimentally34,35 and used for displays.36 The liquid crystal is initially at the cholesteric phase where it has a helical structure whose axis is parallel to the glass substrates. The incident light is scattered and the cell appears milky white. When the applied electric field exceeds \( 10^5 \) V/cm, it will unwind the helix to give an aligned nematic phase. The cell then becomes transparent.

When the voltage is decreased, it has been observed that an intermediate metastable nematic phase could appear.37 In this intermediate phase, the directors near the surfaces remain homeotropic but those in the bulk are slightly tilted. The optical output will therefore exhibit a hysteresis loop. This hysteresis effect can be used for optical storage. Once the voltage is removed completely, the liquid crystal directors return to their initial scattering states within a few ms.

Field-induced Director Axis Reorientation

Field-induced director axis reorientation on aligned nematic and ferroelectric liquid crystals is one of the most common electro-optic effects employed for modulating light. Many alignment methods have been developed for various applications employing nematic liquid crystals. Some examples include 90° twist,38 homeotropic (also called perpendicular) alignment,39 45° twist,40 \( \pi \)-cell,41 and a variety of supertwist cells.42–46 Each alignment exhibits its own unique features and also drawbacks.

Laser-induced Phase Change

Laser-induced phase change has been observed in cholesteric47 and smectic-A48 liquid crystals. In the case of a smectic-A liquid crystal, an IR laser (e.g., laser diode) is focused onto a small spot of the cell. Due to absorption, the irradiated area is heated over the smectic-nematic phase transition temperature. During the cooling process, this area may turn to the scattering or well-aligned nonscattering state, depending on the cooling rate.
and the applied electric field. Slow cooling permits the disordered molecules to reorganize themselves into the initial uniform alignment state which does not scatter light. Application of a sufficiently strong field during cooling also helps such a realignment process. Since the spot size of the writing laser can be as small as about 5 μm, this laser-induced phase change effect can be utilized to construct a light valve with a very high resolution. However, the laser addressing time is relatively slow; it is on the order of seconds, depending on the number of pixels involved.

Light Scattering by Micron-sized Droplets

Micron-sized liquid crystal droplets dispersed in a polymer matrix produce significant light scatterings in the visible spectral region. The refractive index mismatch between the liquid crystal droplets and the host polymer is the physical mechanism responsible for such light scattering in the voltage-off state. At the voltage-on state, the droplets are reoriented along the field direction. The ordinary refractive index of the LC matches the index of the polymer. As a result, little scattering takes place and most of the light is transmitted. One advantage of this type of electro-optical modulation is that it does not require polarized light (i.e., no polarizer is needed). Therefore, optical efficiency is greatly enhanced.

14.5 ELECTRO-OPTICS OF NEMATIC LIQUID CRYSTALS

Field-induced Reorientational Effects

The appearance of liquid crystal molecules without alignment, such as in a bottle, is often milky. They scatter light strongly due to the fluctuations of liquid crystal clusters. A 5-mm-thick liquid crystal cluster in the mesogenic phase is enough to block out the transmission of a visible light. However, at the isotropic state, light scattering is suppressed dramatically and the liquid crystal clusters become clear. For realizing useful electro-optic effect, LC has to be aligned. Three basic alignments in nematic LCs have been developed and widely used for application. They are parallel, perpendicular, and twist alignments.

Parallel Alignments. In the parallel alignment, the directors in the front and back substrates are parallel. These substrates are first coated with an electrically conductive but optically transparent metallic film, such as indium-tin-oxide (ITO), and then deposited with a thin SiO₂ layer to create microgrooves for aligning LC molecules. The oblique-angle evaporation method produces a high-quality alignment, well-defined pretilt angle.

Gently rubbing the ITO surface can produce microgrooves as well. The anchoring energy of the rubbed surfaces is found to be larger than the SiO₂ evaporation method. Simplicity is the major advantage of the rubbing method. This technique has been widely employed for fabricating large panel LC devices. However, some problems of the rubbing technique exist, such as possible substrate contamination, creation of static charges, and variations of pretilt angle from substrate to substrate.

When a linearly polarized light impinges in a parallel-aligned cell, if the polarization axis is parallel (θ = 0°) to the LC director, a pure phase modulation is achieved because the light behaves as an extraordinary ray. On the other hand, if θ = 45°, then phase retardation occurs due to the different propagating speed of the extraordinary and ordinary rays in the LC medium. The phase retardation (δ) is determined by: (1) LC layer
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thickness \(d\); (2) effective LC birefringence, \(\Delta n(V, T, \lambda)\), which is dependent on the applied voltage, temperature, and wavelength \(\lambda\) of the incident light; and (3) wavelength as:

\[
\delta(V, T, \lambda) = \frac{2\pi d \Delta n(V, T, \lambda)}{\lambda}
\]

At \(V = 0\), \(\Delta n (= n_c - n_o)\) has its maximum value; so does \(\delta\). In the \(V > V_o\) regime, the effective birefringence decreases sharply as voltage increases, and then gradually saturates. The slope depends on the elastic constants, dielectric constants, and refractive indices of the LC material. At \(V \gg V_o\) regime, basically all the bulk directors are aligned by the field to be perpendicular to the substrates, except the boundary layers. Thus, further increase in voltage only causes a small change in the orientation of boundary layers. In this regime, the effective birefringence is inversely proportional to the applied voltage.

The advantage of operating an LC device at low voltage regime is that a large phase change can be obtained merely with a small voltage swing. However, the directors’ relaxation time is slow. On the other hand, at high voltage regime the phase change is inert to the applied voltage, but the response time is much faster.

The transmission of a parallel cell is very sensitive to wavelength because the phase retardation depends strongly on wavelength and birefringence, as shown in Eq. (6). Thus, the single parallel-aligned cell is not suitable for color display using a broadband light source. A good contrast can be obtained only when a narrow laser line is used or the device is operated at high voltage regime. On the other hand, the parallel alignment is ideal for pure phase modulation and tunable phase retardation plate.

**Perpendicular (or Homeotropic) Alignment.** Perpendicular alignment is also known as homeotropic alignment. In such an alignment, LC directors are perpendicular to the substrate surfaces except for a small pretilt angle. Similar to parallel alignment, perpendicular alignment can be used for both pure phase modulation and phase retardation. But there exist three major differences between these two alignments. (1) For realizing useful electro-optical effects of a parallel-aligned cell, the LCs must have a positive dielectric anisotropy. But for a perpendicular alignment, LCs with negative dielectric anisotropy \((\varepsilon_r < \varepsilon_s)\) are required. (2) For achieving a stable perpendicular alignment, a certain alignment surfactant, such as DMOAP or alcohol, is needed. But for a parallel alignment, rubbed surfaces work well. (3) The voltage-off state of a parallel-aligned cell between crossed polarizers is dependent on the phase retardation of the cell and is normally bright unless \(\delta\) happens to be \((2m + 1)\pi\). But for a perpendicular-aligned cell, this voltage-off state is dark. Moreover, this dark state is independent of LC thickness, LC birefringence, wavelength, and temperature.

Contrast ratio as high as \(10^5 : 1\) for a laser beam at normal incidence has been observed in uniformly aligned perpendicular cells. For a broadband light, a contrast exceeding \(200 : 1\) is easily achievable. However, the transmission of the voltage-off state slightly increases and contrast ratio decreases gradually as the incident angle tilts away from normal. A typical viewing angle of a homeotropic LC cell is about \(\pm 30^\circ\) in the horizontal direction.

**Twist Alignment.** In the twist alignment, both substrates are treated similarly to those of parallel alignment except that the back substrate is twisted with an angle. In order to realize the useful electro-optical effect, the positive nematic LC material has to be employed. This type of LC cell is abbreviated as a TN cell, standing for the twist nematic. For the twist angle greater than \(90^\circ\), the cell bears a special name as supertwist nematic or STN. STN and TN are widely employed in the passive-matrix and active-matrix liquid crystal displays, respectively.

When a linearly polarized light traverses through a \(90^\circ\) TN cell, the plane of polarization follows the twist of the LC directors if the Mauguin’s condition is satisfied; i.e., \(d\Delta n \gg \lambda\). Under this circumstance, the output beam remains linearly polarized except that its polarization axis is rotated by \(90^\circ\). Thus, for a normally dark (no transmission at
V = 0) display using a 90° TN cell, the analyzer is set to be parallel to the polarizer. But when the applied voltage exceeds a threshold called optical threshold, the polarization guiding effect is interrupted and the light leaks through the analyzer.

Several twist alignments have been developed according to the twist angle, such as 45°, 90° twist, super birefringence effect (SBE, 270° twist), optical mode interference (OMI, 180° twist), and supertwist nematic with twist angle ranging from 200 to 270°. For high-density information display applications using the multiplexing addressing technique, the sharp voltage-dependent optical transmission curve is highly desirable. The regular TN cells with twist angle smaller than 90° do not meet this challenge. Thus, several STN configurations have been developed for steepening the electro-optical transmission.

In the 90° TN cell, the director of the back surface is twisted 90° with respect to the front surface. The 90° TN cell is normally used in the transmissive mode. Figure 9 shows the directors’ configurations of the 90° TN cell under three conditions: (1) V < Vt, (2) V > Vt, and (3) V > Vt. In Fig. 9, φ and θ represent the tilt and twist angles of the LC directors, respectively. At a V not too far above Vt, the LC directors tilt collectively (but reserve the twist configuration) along the field direction. Thus, the output beam is modulated in phase only, but not in amplitude. However, as voltage further increases, the twist continuity of the LC directors is interrupted and its polarization guiding effect is broken. As a result, light leaks through the analyzer.

Optical Transmission of LC Cells

Optical transmission of an LC cell sandwiched between two polarizers is a fundamentally important subject for electro-optic application. Extensive efforts have been devoted to obtaining the analytical expressions and then optimizing the performance of various display modes. Three approaches are commonly employed: (1) the geometrical-optics approach, (2) the 4 × 4 matrix method, and (3) the extended Jones matrix method. Both 4 × 4 matrix and extended Jones matrix methods are powerful in treating the oblique incidence which is particularly important for improving the viewing angle of the LC devices.

The generalized geometrical optics approach is developed for solving the optical transmission of a general TN cell with an arbitrary twist angle and for the double-layered
The geometry of an LC cell and polarizer orientation is depicted in Fig. 10. Here, a TN cell with thickness $d$ is confined between the planes $Z_1 = 0$ and $Z_2 = d$ in a cartesian coordinate system. Both substrate surfaces are treated with alignment layers which give a uniform pretilt angle $\phi_S$. $\phi_S$ is typically 1 to 5° for the twist angle $\theta \approx 240°$, and $-20°$ for the $270°$ STN. This pretilt angle is necessary in order to avoid domain formation during director rotation. As shown in Fig. 10, the LC director at the entrance surface is directed along the $x$-axis and twisted by an angle $\theta$ at the exit surface. The entrance and exit polarizers are arranged at angles $\phi_{ent}$ and $\phi_{exit}$ with respect to the LC director, respectively.

After having considered the propagation of both ordinary and extraordinary rays in the LC medium, Ong\textsuperscript{59} has derived the light transmission for an arbitrary twist LC cell under normal incidence condition:

$$T = \cos^2(\theta - \phi_{exit} + \phi_{ent}) + \sin^2(\theta \sqrt{1 + u}) \sin 2(\theta - \phi_{exit}) \sin 2\phi_{exit}$$

$$+ \frac{1}{2\sqrt{1 + u}} \sin (2\theta \sqrt{1 + u}) \sin 2(\theta - \phi_{exit} + \phi_{ent})$$

$$- \frac{1}{1 + u} \sin^2(\theta \sqrt{1 + u}) \cos 2(\theta - \phi_{exit}) \cos 2\phi_{exit}$$

where

$$u = (\pi d/\theta \lambda)(n_e/\sqrt{1 + v \sin^2 \phi_S})$$

and $v = (n_e/n_o)^2 - 1$. From this general expression, the transmission (under different polarizer-analyzer orientations) of parallel-aligned ($\theta = 0°$) TN and STN cells can be calculated.

**LC Directors’ Response to External Field**

To solve the steady state directors’ distribution, the total free energy needs to be minimized. The total free energy for nematics consists of two parts: (1) dielectric free energy which originates from the interaction between the applied field and anisotropic LC molecules, and (2) elastic free energy which originates from the elastic deformation. On the other hand, to derive the dynamic response of the LC directors, the elastic torque and electric field-induced torque need to balance with the viscous torque. Let us illustrate the detailed procedures using a parallel-aligned cell as an example. The treatment of a perpendicular cell is similar to that of a parallel one except for some parameter changes. Once the directors’ distribution is obtained, the voltage-dependent phase retardation and capacitance changes can be calculated.
Directors’ Distribution. Consider a parallel aligned nematic LC cell in which the directors are along the \( x \) axis and the two bounding surfaces are at \( Z_1 = 0 \) and \( Z_2 = d \). When the applied voltage \( V \) (along the \( z \)-axis) exceeds the Freedericksz transition threshold \( V_{th} \), the LC undergoes an elastic deformation. The directors then tilt in the \( xz \) plane, the amount of tilt \( \phi(z) \) being a function of the distance from the aligning surface; \( \phi(z) \) has a maximum value \( \phi_m \) at \( Z = d/2 \) and \( \phi(z) = 0 \) at boundaries.

The static state directors’ distribution is described by the Oseen-Frank equation:

\[
\begin{align*}
\frac{d^2 \phi}{dz^2} + (K_{33} - K_{11}) \sin \phi \cos \phi \frac{d\phi}{dz} &= \varepsilon_0 \Delta \varepsilon E^2 \sin \phi \cos \phi \quad (9)
\end{align*}
\]

where \( K_{11}, K_{22}, \) and \( K_{33} \) stand for splay, twist, and bend elastic constant, respectively. After some algebra and introducing two new parameters, \( \sin \phi = \sin \phi_m \sin \phi \) and \( \eta = \sin^2 \phi_m \), the following important equations are obtained:

\[
\begin{align*}
\frac{V}{V_{th}} &= \frac{2}{\pi} \sqrt{\frac{1}{1 + 2\gamma}} \int_0^{\pi/2} \frac{1 + \kappa \eta \sin^2 \psi}{\sqrt{(1 + \gamma \eta \sin^2 \psi)(1 - \eta \sin^2 \psi)}} d\psi \quad (10)
\end{align*}
\]

and

\[
\begin{align*}
\frac{2z}{d} &= \int_0^{\pi/2} \left[ \frac{(1 + \kappa \eta \sin^2 \psi)(1 + \gamma \eta \sin^2 \psi)}{1 - \eta \sin^2 \psi} \right]^{1/2} d\psi \\
&= \frac{1}{\sin^{-1}(\sinh\sqrt{\gamma})} \left[ \frac{(1 + \kappa \eta \sin^2 \psi)(1 + \gamma \eta \sin^2 \psi)}{1 - \eta \sin^2 \psi} \right]^{1/2} d\psi \quad (11)
\end{align*}
\]

where \( V_{th} \) is the threshold voltage given by \( V_{th} = \pi[K_{11}/\varepsilon_0 \Delta \varepsilon]^{1/2} \), \( \gamma = \varepsilon_0/\varepsilon_\perp - 1 \), and \( \kappa = K_{11}/K_{33} - 1 \).

In principle, knowing the liquid crystal material constants, one can use Eq. (10) in an iterative fashion to evaluate \( \phi_m \) for a given applied voltage. Once \( \phi_m \) is obtained, the complete profile of \( \phi(z) \) can be calculated from Eq. (11). Figure 11 shows the voltage-dependent directors’ distribution of a parallel-aligned LC cell. At \( V = 2V_{th} \), \( \phi_m \approx 60^\circ \). But at \( V = 4V_{th} \), \( \phi_m > 85^\circ \). Thus, voltage-dependent phase change is sensitive at

![FIGURE 11](image-url) Director’s distribution of a parallel-aligned LC cell for various normalized voltage \((V/V_{th})\).
low voltage regime and gradually saturates as voltage increases. At high voltage regime, the central layers all reach their maximum tilt angle; only the surface layers tilt further. Thus, a small phase change is obtained with a large voltage increase.

**Dynamic Response.** In a static deformation of LC directors, the Oseen-Frank equation describes the balance between an elastic torque and an electric torque exerted by the applied field. But in the dynamic response, the viscous torque which opposes the directors’ rotation has to be included. The most general treatment of the dynamic of LC directors is described by the Erickson-Leslie equation:

$$
\frac{\partial}{\partial z} \left[ \left( K_{11} \cos^2 \phi + K_{33} \sin^2 \phi \right) \frac{\partial \phi}{\partial z} \right] + (K_{33} - K_{11}) \sin \phi \cos \phi \left( \frac{\partial \phi}{\partial z} \right)^2
$$

$$
+ (\alpha_2 \sin^2 \phi - \alpha_3 \cos^2 \phi) \frac{\partial v}{\partial z} + \varepsilon_0 \Delta \varepsilon E^2 \sin \phi \cos \phi = \gamma_1 \frac{\partial \phi}{\partial t} + I \frac{\partial^2 \phi}{\partial t^2}
$$

where $\phi$ is the deformation angle, $\alpha_{2,3}$ are Leslie viscosity coefficients, $v$ is the flow velocity, $\gamma_1 = \alpha_3 - \alpha_2$ is the rotational viscosity, and $I$ is the inertia of the LC directors. The Erickson-Leslie equation can be applied to both parallel, perpendicular, and twist alignments. We will deal with the simplest case, which is parallel alignment.

Neglecting the backflow and inertial effects in Eq. (12), the dynamic response of parallel-aligned LC directors is described as:

$$(K_{11} \cos^2 \phi + K_{33} \sin^2 \phi) \frac{\partial^2 \phi}{\partial z^2} + (K_{33} - K_{11}) \sin \phi \cos \phi \left( \frac{\partial \phi}{\partial z} \right)^2
$$

$$
+ \varepsilon_0 \Delta \varepsilon E^2 \sin \phi \cos \phi = \gamma_1 \frac{\partial \phi}{\partial t}
$$

The rise and decay times of the LC directors are described as:

$$
\tau_{\text{rise}} = \frac{\tau_o}{[(V/V_b)^2 - 1]}
$$

$$
\tau_{\text{decay}} = \frac{\tau_o}{[(V_b/V)^2 - 1]}
$$

where $\tau_o = \gamma_1 d^2/K_{11} \pi^2$ is the free relaxation time, $V_b$ is the bias voltage of the LC cell, or final state of the relaxation. The absolute value in the denominator indicates that this formula is valid no matter if $V_b$ is above or below $V_{th}$. It should be mentioned that the response times appearing in Eqs. (26) and (27) are directors’ responses to the external field, but not the optical responses. Several methods have been developed for improving the response times of nematic LC devices. Among them, dual frequency effect, bias voltage effect, transient nematic effect, temperature effect, Fabry-Perot effect, and molecular engineering method have been proven effective.

### 14.6 Electro-optics of Polymer-Dispersed Liquid Crystals

A simple diagram illustrating the electro-optical effect of a polymer-dispersed liquid crystal (PDLC) shutter is shown in Figs. 12a and 12b for the off and on states, respectively. Suppose the polymer matrix material is optically isotropic and has refractive index $n_p$. The LC directors within the droplets are determined by the polymer-LC interaction at each droplet boundary. They have no preferred orientation but vary nearly randomly from droplet to droplet in the absence of external field. The index mismatch ($n_p > n_d$) between the LC droplets and the host polymer results in light scattering. Because the cell thickness
(\sim 10 \mu m) is much larger than the droplet size (\sim 0.5 \mu m), the incident light will be scattered many times before emerging from the film. Thus, little light is transmitted through the cell. The degree of the off-state scattering depends on the size, birefringence, and concentration of the liquid crystalline droplets, and the film thickness. When the applied field is sufficiently strong, the directors in the droplets are reoriented along the field direction. Its effective refractive index is equal to \( n_p \) which is similar to the index of the polymer, \( n_p \). Thus, the normally incident light acts as the ordinary ray and transmits through the cell. The PDLC film becomes clear. This mode of operation is called normal mode. Reversed mode operation has been demonstrated recently using nematic liquid crystals and gels containing cholesteric liquid crystals.

Physical Mechanisms

Optical transmission of a nonabsorbing PDLC film depends on the scattering properties of the micron-sized droplets and their distribution in space. The light scattering induced by the micron-sized nematic droplets is much more significant than that from the directors’ fluctuation. To find an exact solution for the optical transmission of a PDLC system is a difficult task because the droplet size is comparable to the wavelength in the visible region. Thus, a certain approximation (depending on the relative dimension of the droplet size as compared to the wavelength of the incident light) has to be taken. If the object size is much smaller than the wavelength and its optical anisotropy is not very large, the Rayleigh-Gans approximation can be employed and the total scattering cross section is found to be proportional to \( \lambda^{-2} \). On the other hand, if the object size is larger than the wavelength, the anomalous diffraction approach can be used and the total cross section is proportional to \( \lambda^{-2} \). If the object size is very large, the geometrical optics approach can be taken.

Dynamic Response of PDLC

The dynamic response of a PDLC film has been solved by Wu et al., and the rise and decay times are shown as follows:

\[
\tau_{\text{on}} = \frac{\gamma I}{\varepsilon_0 \Delta \varepsilon E^2 + K (I^2 - 1) / \alpha^2} \tag{16}
\]

\[
\tau_{\text{off}} = \frac{\gamma I a^2}{K (I^2 - 1)} \tag{17}
\]
The rise time can be shortened by increasing the applied voltage. However, the relaxation time is proportional to the viscosity, droplet size \( a \), and effective elastic constant \( K \) in the same way as a nematic LC cell, except that the aspect ratio also plays an important role. For a PDLC that consists of E-7 LC droplets, a slightly shaped droplet of \( l = 1.1 \) leads to \( \tau_{\text{off}} \approx 20 \text{ ms} \). Equation (17) predicts that the more elongated LC droplets yield faster rise and decay times. A simple way to obtain elongated droplets is to shear the PDLC film during droplet formation.

**Photoactivated PDLC Light Valve**

A reflective-mode, photoactivated PDLC light valve has been demonstrated\(^{76}\) and results are shown in Fig. 13. This device is particularly useful for projection TV display. The photoconductor used is BSO (Bi\(_{12}\)SiO\(_{20}\)), whose photosensitivity is in the visible region.\(^{77}\) In the beginning, an external voltage \( V_b = 35 \text{ V}_{\text{rms}} \) is applied to the PDLC light valve. Due to the high impedance of the photoconductor, little voltage drops across the PDLC layer. The PDLC is in its scattering state; very little light is transmitted to the detector. As the photoconductor is illuminated by a wavelength \( \lambda = 457.9 \text{ nm} \) from an Ar\(^+\) laser which is within the bandgap of BSO, the photogenerated carriers cause the impedance of the photoconductor to decrease. As a result, more voltage drops across the PDLC film, which increases the transmittance of the readout light. The response time of the PDLC light valve is about 30 ms. Note that this light-scattering mechanism does not require a polarizer and, therefore, high brightness can be obtained, which is important for large screen display.

**14.7 ELECTRO-OPTICS OF FERROELECTRIC LIQUID CRYSTALS**

**Surface-stabilized FLCs**

The bookshelf geometry of the surface-stabilized ferroelectric liquid crystal cell\(^{78,79}\) is sketched in Fig. 14. The molecules of Sm-C* LC form a layered structure located in the
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FIGURE 14 The coordinate system used for describing the dynamic response of a surface-stabilized FLC cell.79

$x$-$y$ plane. The LC director $\mathbf{n}$ tilts away from the layer normal (z axis) by a constant angle $\theta$. The projection of $\mathbf{n}$ onto the layer plane is defined as the C-director and $\phi$ is the azimuthal angle between the C-director and the $x$ axis. The spontaneous polarization $P_s$ is located in the $x$-$y$ plane but perpendicular to both $\mathbf{n}$ and C-directors. The electric field is applied along the $y$ axis and the sample is assumed to be uniform in the $x$ and $z$ axes without any deformation.

Using the similar treatment of free energy, Nakagawa et al.80 have derived the dynamic response for the surface-stabilized ferroelectric LC:

$$
\gamma_1 \frac{\partial \phi}{\partial t} = A(1 + \sin^2 \theta) \left( \frac{\partial^2 \phi}{\partial y^2} \right) + \frac{v}{2} A \sin 2\phi \left( \frac{\partial \phi}{\partial y} \right)^2 + \frac{e}{2} \varepsilon_x E^2 \sin 2\phi + P_s E \sin \phi
$$  \hspace{1cm} (18)

where $A = K_{11} \sin^2 \theta$, $v = (K_{22}/K_{11}) \cos^2 \theta + (K_{33}/K_{11}) \sin^2 \theta - 1$, and $e = (e_i/e_x - 1) \sin^2 \theta$.

In the one elastic constant approximation ($K_{11} = K_{22} = K_{33} = K$), Eq. (18) is greatly simplified:

$$
K \sin^2 \theta \frac{\partial^2 \phi}{\partial y^2} + \frac{e}{2} \varepsilon_x E^2 \sin 2\phi + P_s E \sin \phi = \gamma_1 \frac{\partial \phi}{\partial t}
$$  \hspace{1cm} (19)

For comparison, the magnitude of each term is estimated as follows. For a typical FLC material and device, $K \approx 10^{-11} \text{N}$, $\theta \approx 20^\circ$, $\varepsilon_x \approx 10^{-11} \text{F/m}$, $E \approx 10^6 \text{V/m}$, and $P_s \approx 10^{-5} \text{C/m}^2$. The value $e$ can be positive or negative depending on the dielectric anisotropy of the LC. If an FLC has its $\varepsilon_i = \varepsilon_x$, then $e = 0$ and the dielectric (second) term in Eq. (19) can be neglected. Under this condition, Eq. (19) is reduced to that derived by Yamada et al.81 On the other hand, if $e$ is not too small, the magnitude of the dielectric term may be comparable to that of the spontaneous polarization (third) term. The elastic term becomes the smallest among the three and may be neglected. Under this assumption, Eq. (19) is then reduced to that derived by Xue et al.:82,83

$$
\frac{1}{2} \Delta \varepsilon E^2 \sin 2\phi + P_s E \sin \phi = \gamma_1 \frac{\partial \phi}{\partial t}
$$  \hspace{1cm} (20)

Xue et al.83 derived the following analytical solution for Eq. (20):

$$
t = \frac{1}{\tau} = \frac{1 - a^2}{1 - a^2 \left\{ \tan \left( \frac{\phi/2}{2} \right) + a \tan \left( \frac{\phi_0/2}{2} \right) \right\} + a \left( 1 + a \cos \phi_0 \right) \sin \phi_0}
$$  \hspace{1cm} (21)

where $\tau = \gamma_1/P_s E$, $\phi_0 = \phi(t = 0)$, and $a = \Delta \varepsilon E \sin^2 \theta/(2P_s)$. 
In practical application, an SSFLC film is arranged such that the input light polarization is parallel to the director which is one of the bistable states—say, “up” state. The analyzer is crossed with respect to the polarizer. Thus, no light is transmitted and a good dark state is obtained. This dark state is independent of the incident wavelength. When the sign of the electric field is reversed, the director is changed from the up state to the down state (the total rotation angle is $2\theta$), and some light leaks through the analyzer. The normalized transmission is proportional to the tilt angle $\theta$ and phase retardation $\delta$ as

$$T'_x = \sin^2(4\theta) \sin^2(\delta/2)$$  \hspace{1cm} (22)

As seen from Eq. (22), the optimal transmission occurs at $\theta = 22.5^\circ$ and $\delta = \pi$; i.e., the FLC film acts like a half-wave phase retardation plate. In order to achieve a uniform rotation, the SSFLC layer thickness is often limited to about 2 $\mu$m. This bright state transmission is dependent on the cell thickness, wavelength, and the birefringence of the FLC mixture employed. The response time of such an FLC cell is proportional to $\gamma_1/PE$ and ranges from 1 to 100 $\mu$s. The lower rotational viscosity, larger spontaneous polarization, and higher switching field are helpful in improving the response times of the FLC devices.

**Deformed Helix Ferroelectric (DHF) Effect**

The director configuration of the DHF LC cell is shown in Fig. 15. The applied dc electric voltage deforms the helix of the FLC. As a result, the effective birefringence of the cell is changed and the transmitted light modulated. The DHF effect is complementary to the SSF effect in many ways. The basic features of the DHF effect are summarized as follows: (1) it uses short (~0.35 $\mu$m) pitch mixtures; (2) its surface anchoring energy is kept weak, just enough to generate the so-called “bookshelf structure”; (3) the field-induced tilt angle is linear to the applied voltage; (4) it has no inherent bistability; (5) it exhibits no optical threshold voltage; (6) it possesses greyscale; and (7) it operates at much lower voltage, and with response time shorter than the SSFLC devices. However, two problems remain to be overcome before the DHF effect can be used for displays: (1) the hysteresis effect, and (2) the relatively low contrast ratio. The hysteresis effect is more pronounced at higher voltages. This hysteresis makes the precise control of greyscale difficult. The contrast depends strongly on how the cell is prepared. The sheared cell seems to provide a reasonably good LC alignment. A contrast ratio of 40 : 1 has been reported.

**FIGURE 15** Cell configuration of the deformed helix effect. The incident light is normal to the cell. $L$ = cell gap; $p$ = helical pitch; $Z$ = helical direction; $n$ = long axis of the FLC; and $P$ = spontaneous polarization vector.96
Soft-mode FLCs

There are similarities and differences between the soft-mode and the surface-stabilized-mode ferroelectric effects. The similarities include the following: (1) Both effects are linear to the applied electric field. Thus, they are sensitive to the polarity of the applied field. This is different from the nematic, which is sensitive to the square of the field. (2) Both effects have fast response times. The major differences can be found in the following areas: (1) SSFLCs use changes in the azimuthal angle \( \phi \) around the tilt cone; SMFLCs use changes in tilt \( \theta \). For SSFLC, \( \theta \) remains constant and \( \phi \) varies (called Goldstone mode). But for SMFLC, \( \phi \) remains constant and \( \theta \) varies (called soft mode). In general, the electric field-induced deflection angle in SMFLCs is small so that the associated optical change is small. (2) SSFLCs exhibit bistability, but SMFLCs show continuously controllable intensity change. (3) The SMFLCs employ smectic A* phase, but SSFLCs use smectic C* phase. Thus, the uniform alignment is much easier to obtain for SMFLCs than those for SSFLCs. The LC layer thickness in SSFLCs is usually limited to 2 \( \mu \)m in order to achieve uniform surface-stabilized states. But in SMFLCs, this requirement is greatly relieved. Good alignment can still be obtained even if the LC layer is as thick as 10 \( \mu \)m.

The experimental apparatus for realizing the soft-mode FLC effect is sketched as shown in Fig. 16. In the small angle approximation, the tilt angle \( \theta \) is obtained as

\[
\theta = e_c E \tag{23a}
\]

\[
e_c = \frac{\mu}{\alpha(T - T_c)} \tag{23b}
\]

where \( e_c \) is called the electroclinic coefficient, \( \mu \) is the structure coefficient which is
equivalent to the dipole moment per unit volume for unit tilt angle, and \(\alpha\) is a proportionality constant. From Eq. (23b), \(\varepsilon\) diverges when \(T\) approaches the Sm-A* to Sm-C* transition. At a given temperature, the field-induced tilt angle is linearly proportional to the applied electric field.

The decay time of the SMFLC cell has been derived\(^{91}\) and expressed as

\[
\tau = \frac{\gamma_0}{\alpha(T - T_c)}
\]  

(24)

To the first-order approximation, the response time is independent of the applied electric field. This prediction is also validated experimentally.\(^{91}\) However, at near the smectic A-C* transition, the response time is dependent on temperature and the applied field.\(^{92}\) As temperature approaches \(T_c\), \(\tau \sim E^{-2X}\) with \(1/3 \leq X \leq 1\). Near transition, \(\tau\) decreases rapidly as \(E\) increases. Far from the Sm A Sm-c* transition, \(\tau\) is weakly dependent on \(E\).

Response time of about 1 \(\mu\)s has been demonstrated although the modulation efficiency is low due to the small tilt angle. In the smectic-A* phase, it is difficult to obtain an induced tilt angle much greater than \(10^\circ\) unless one works near the phase transition. In this regime, the response time is comparatively slow and strongly dependent on the temperature. From Eq. (22), the maximum optical transmission for \(\theta = 10^\circ\) is calculated to be about 40 percent. One way to improve the optical transmission efficiency is to use two or more SMFLC cells in series. For example, if one arranges two electroclinic cells together and operates with reversed voltages, one can achieve a 100 percent modulation efficiency with tilt angle \(\theta = 11.25^\circ\) and phase retardation \(\delta = \pi\). Under these circumstances, the response times remain the same as those of the single cell, but the optical modulation is greatly enhanced.

### 14.8 CONCLUSION

Liquid crystals are a unique electro-optic medium with spectral transmission spanning from UV, through visible and infrared to the microwave region. In the UV region, some electronic transitions take place. These absorptions determine the refractive index dispersions of the liquid crystals. Various physical mechanisms and devices, such as the dynamic scattering mode, the guest-host effect, the field-induced cholesteric-nematic phase transition, the field-induced director reorientation effect, light scattering induced by the micron-sized liquid crystal droplets, and the thermo-optic effect, have been developed for modulating light. The response times of the nematics are typically in the 10- to 100-ms range. For high-information-content displays using nematics, the device response time is marginal and further improvement is needed.

To achieve a faster response time, ferroelectric liquid crystals can be considered. Several electro-optic effects based on the ferroelectric liquid crystals have been developed. Among them, the surface-stabilization effect, deformed helix effect, and electroclinic effect hold great promise. The surface-stabilized FLCs exhibit bistability and fast response times. Thus, they are suitable for flat panel displays which don’t require greyscale. On the other hand, both electronic effects and deformed helix exhibit greyscale and fast response times. The low transmission of the electroclinic FLC cell can be overcome by using two cells in series. However, the electroclinic coefficient is quite sensitive to temperature. Thus, the operation temperature of the device needs to be controlled precisely. Although the deformed helix effect is promising for display application, its hysteresis effect needs to be minimized and its contrast ratio improved.
5. For a review, see P. S. Pershan, *Physics Today*, May 1982, p. 34.
27. S. T. Wu and C. S. Wu, *Liq. Cryst.* 8:171 (1990). Seven commonly used models (see the references therein) have been compared in this paper.