Confinement and Modal Gain in Dielectric Waveguides

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Abstract—**Two exact expressions are derived for the effective mode indices of dielectric slab waveguides with complex refractive indices. One is for TE modes, the other for TM modes. The identities are valid for any guided mode of any dielectric slab waveguide and can be succesfully employed to check the accuracy of mode solvers. They explain why TE gain can be much greater than TM gain even when the confinement factors are comparable. Also, it is shown that an often used approximation for the TM gain is unreliable under practical conditions. A correct approximation is given.**

I. Introduction

In a recent study of active slab waveguides [1] it was found that for certain configurations the gain (in dB/μm) for TE modes can be twice that for TM modes. Surprisingly, the confinement factors did not differ substantially for the two modes. The confinement factor Γ can be defined as [2]

$$\Gamma \equiv \frac{\int_{\text{active layer}} |\langle S(x_1)\rangle|\, dx_1}{\int_{-\infty}^{\infty} |\langle S(x_1)\rangle|\, dx_1} \tag{1}$$

with $\langle S(x_1)\rangle$ the time-averaged Poynting vector, and with the integral in the numerator over the active layer. An example is given in Fig. 1. For this realistic configuration it is found that the TE_0 gain is 3.83 dB/100 μm, against 2.39 for TM_0. The confinement factors are 44.0% and 40.7%, respectively. Obviously, this large difference in gain cannot be 'explained' by a difference of less than 10% in the confinement factors. The aim of this study is twofold:

1) To clarify the relationship between the confinement factors and the modal gain in TE and TM guided modes, and
2) To show that an often used approximation for the TM gain is invalid for practical configurations.

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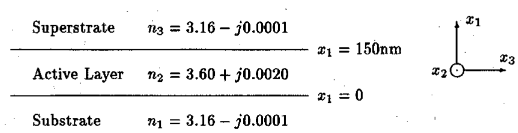

Fig. 1. Example of a symmetric three layer dielectric waveguide. The wavelength in vacuum$\lambda_0 = 1.3$ μm. It was found that Imn_{eff} =9.13293194 $\times 10^{-4}$ for the TE_0 mode, and Imn_{eff} = 5.69870521 $\times 10^{-4}$ for TM_0. The confinement fators in terms of the Poynting vector are 44.0% and 40.7%, respectively.

Planar waveguides are studied, i.e., the permittivity $\epsilon = \epsilon(x_1)$ is taken as piecewise constant (see Fig. 1). The configuration is invariant in the directions of x_2 and x_3. We investigate solutions that represent guided modes that propagate in the positive x_3-direction, i.e., solutions which are of the form

$$\{\hat{\mathbf{E}}, \hat{\mathbf{H}}\}(x_1, x_3; k_3) = \{\tilde{\mathbf{E}}, \tilde{\mathbf{H}}\}(x_1; k_3) \exp[-j(k_3 x_3 - \omega t)] \tag{2}$$

with k_3 the (complex) propagation constant (called β by some authors) of the mode. The effective index of a guided mode is defined as $n_{\text{eff}} = k_3/k_0$, where $k_0 = \omega/c$ is the free-space wavenumber. We derive exact expressions for the effective permittivity $\epsilon_{\text{eff}} = (k_3/k_0)^2$, and approximations for the modal gain coefficient $g_{\text{mod}} = 2k_0 \operatorname{Im}\{n_{\text{eff}}\}$.

The media are assumed to be isotropic, nonconducting, linear and nonmagnetic. Hence, the permeability $\mu = \mu_0$ everywhere. The fields are assumed to be independent of x_2, that is, $\partial_2 = 0$. The Maxwell equations for the steady state for a source-free region now separate into two sets of three relations. One describes TM guided modes, with $\{\tilde{H}_2, \tilde{E}_1, \tilde{E}_3\} \neq 0$. Eliminating $\tilde{E}_1$ and $\tilde{E}_3$ from this set yields the following wave equation for $\tilde{H}_2$

$$[\partial_1^2 - k_3^2 + \omega^2 \epsilon \mu_0]\tilde{H}_2 - \frac{\partial_1 \epsilon}{\epsilon}\partial_1 \tilde{H}_2 = 0. \tag{3}$$

Multiplying this with the complex conjugate of $\tilde{H}_2$ and integrating from minus to plus infinity gives

$$k_3^2 \int_{-\infty}^{\infty} |\tilde{H}_2(x_1)|^2 dx_1 = \int_{-\infty}^{\infty} \tilde{H}_2^* \partial_1^2 \tilde{H}_2 dx_1 + \omega^2 \epsilon_0 \mu_0 \times \int_{-\infty}^{\infty} \epsilon_r |\tilde{H}_2(x_1)|^2 dx_1 - \int_{-\infty}^{\infty} \tilde{H}_2^* \frac{\partial_1 \epsilon}{\epsilon} \partial_1 \tilde{H}_2 dx_1 \tag{4}$$

with $\epsilon = \epsilon_r \epsilon_0$, and ϵ_0 the free-space permittivity. Concentrating on the first term on the right-hand side

$$\int_{-\infty}^{\infty} \tilde{H}_2^* \partial_1^2 \tilde{H}_2 dx_1 = \int_{-\infty}^{\infty} \partial_1 (\tilde{H}_2^* \partial_1 \tilde{H}_2) dx_1 - \int_{-\infty}^{\infty} \partial_1 \tilde{H}_2^* \partial_1 \tilde{H}_2 dx_1, \quad (5)$$

$$= [\tilde{H}_2^* \partial_1 \tilde{H}_2]_{-\infty}^{+\infty} - \int_{-\infty}^{\infty} |\partial_1 \tilde{H}_2|^2 dx_1. \quad (6)$$

For the guided modes that we are considering, both $\tilde{H}_2^*(x_1)$ and $\partial_1 \tilde{H}_2(x_1)$ vanish for $|x_1| \to \infty$, so the first term on the right-hand side of (6) is zero. The second term is positive definite. Hence we conclude that

$$\int_{-\infty}^{\infty} \tilde{H}_2^* \partial_1^2 \tilde{H}_2 dx_1 \in \mathcal{R}. \quad (7)$$

Taking the imaginary part of Eq. (4) thus yields

$$\mathrm{Im}\{k_3^2\} \int_{-\infty}^{\infty} |\tilde{H}_2(x_1)|^2 dx_1 = k_0^2 \int_{-\infty}^{\infty} \mathrm{Im}\{\epsilon_r\} |\tilde{H}_2(x_1)|^2 dx_1 - \mathrm{Im}\left\{ \int_{-\infty}^{\infty} \tilde{H}_2^* \frac{\partial_1 \epsilon}{\epsilon} \partial_1 \tilde{H}_2 dx_1 \right\}, \quad (8)$$

where we used $k_0^2 = \omega^2 \epsilon_0 \mu_0$. Next, we define the TM confinement factor Γ_i^{TM} of the ith layer as

$$\Gamma_i^{\mathrm{TM}} \equiv \frac{\int_i |\tilde{H}_2(x_1)|^2 dx_1}{\int_{-\infty}^{\infty} |\tilde{H}_2(x_1)|^2 dx_1}, \quad (9)$$

where the integral in the numerator is over layer i only. Analyzing the second term of the right-hand side of (8) we notice, since we had assumed the permittivity ϵ to be piecewise continuous, that

$$\frac{\partial_1 \epsilon}{\epsilon} = \partial_1 (\ln \epsilon_r) = \Delta \epsilon_r \delta(x_1 - x_{1,i}) \quad (10)$$

here $x_{1,i}$ stands for the x_1-coordinate of the ith interface between two layers. $\Delta \epsilon_r$ is the discontinuity of the derivative of $\ln \epsilon_r$ at the interface at $x_1 = x_{1,i}$, i.e.,

$$\Delta \epsilon_r (x_{1,i}) \equiv \lim_{x_1 \downarrow x_{1,i}} \ln \epsilon_r(x_1) - \lim_{x_1 \uparrow x_{1,i}} \ln \epsilon_r(x_1). \quad (11)$$

The field $\tilde{H}_2^*(x_1)$ is tangential and hence continuous at each interface. The last factor, $\partial_1 \tilde{H}_2$ is discontinuous at the interface levels, as can be seen from the Maxwell equation

$$\partial_1 \tilde{H}_2 = j\omega\epsilon \tilde{E}_3. \quad (12)$$

If we introduce the average value as

$$2\overline{\partial_1 H_2}(x_{1,i}) \equiv \lim_{x_1 \downarrow x_{1,i}} \partial_1 \tilde{H}_2(x_1) + \lim_{x_1 \uparrow x_{1,i}} \partial_1 \tilde{H}_2(x_1), \quad (13)$$

and substitute (9), (10), and (13) into (8), we finally arrive at

$$\mathrm{Im}\{\epsilon_{\mathrm{eff}}\} = \sum_i \Gamma_i^{\mathrm{TM}} \mathrm{Im}\{\epsilon_{r,i}\} - I^{-1} \sum_{x_{1,i}} \mathrm{Im}\{\tilde{H}_2^* \Delta \epsilon_r \overline{\partial_1 H_2}\} \quad (14)$$

where the abbreviation I stands for

$$I \equiv k_0^2 \int_{-\infty}^{\infty} |\tilde{H}_2(x_1)|^2 dx_1. \quad (15)$$

Incidentally, from (12) it is seen that the second sum in (14) can also be expressed in field quantities that are continuous. Notice that the first sum is over all layers that make up the waveguide, whereas the second sum is over all interfaces. Equation (14) is an exact expression. That is, any TM guided mode solution (cf. 2) must have a propagation constant k_3 such that $\mathrm{Im}\{(k_3/k_0)^2\}$ satisfies (14).

Next we study TE solutions. For these $\{\tilde{E}_2, \tilde{H}_1, \tilde{H}_3\} \neq 0$. Eliminating $\tilde{H}_1$ and $\tilde{H}_3$ from the Maxwell equations yields the following (Helmholtz) wave equation for $\tilde{E}_2$

$$[\partial_1^2 - k_3^2 + \omega^2 \epsilon \mu_0] \, \tilde{E}_2 = 0. \quad (16)$$

Apart from one term that is missing, this is identical with (3) for $\tilde{H}_2$ in the TM case. Notice that if we had allowed for a varying permeability μ, we would have introduced a term containing $\partial_1 \mu / \mu$. However, the technologically relevant semiconductor materials are nonmagnetic. Therefore, in practice, the TE fields satisfy a wave equation that differs from that for the TM fields. As we shall see, it is this asymmetry that explains why, with similar confinement factors, the gain for TE modes can be, e.g., twice as great as that for TM modes.

We now proceed in a similar manner as for the TM modes. That is, we multiply (16) by the complex conjugate of $\tilde{E}_2$, and integrate the result from minus to plus infinity. This gives us

$$k_3^2 \int_{-\infty}^{\infty} |\tilde{E}_2(x_1)|^2 dx_1 = \int_{-\infty}^{\infty} \tilde{E}_2^* \partial_1^2 \tilde{E}_2 dx_1 + \omega^2 \epsilon_0 \mu_0 \int_{-\infty}^{\infty} \epsilon_r |\tilde{E}_2(x_1)|^2 dx_1. \quad (17)$$

Incidentally, (4) and (17) are variational expressions (see e.g., [2], [3]), which can be used to optimize initial guesses for the field distribution [4]. Just as in (5), it follows that

$$\int_{-\infty}^{\infty} \tilde{E}_2^* \partial_1^2 \tilde{E}_2 dx_1 \in \mathcal{R}. \quad (18)$$

So, taking the imaginary part of (17) yields

$$\mathrm{Im}\{\epsilon_{\mathrm{eff}}\} = \sum_i \Gamma_i^{\mathrm{TE}} \mathrm{Im}\{\epsilon_{r,i}\} \quad (19)$$

where we have now defined

$$\Gamma_i^{\mathrm{TE}} \equiv \frac{\int_i |\tilde{E}_2(x_1)|^2 dx_1}{\int_{-\infty}^{\infty} |\tilde{E}_2(x_1)|^2 dx_1}. \quad (20)$$

Notice, in contrast with (14) for the TM case, that the identity (19) for TE polarization consists of a sum over the layers only. Another difference is that the confinement factors Γ for both cases are in terms of $|\tilde{H}_2|^2$ and $|\tilde{E}_2|^2$, respectively. Only the latter can easily be expressed in terms of the Poynting vector. Equation (19) is an exact relation that must be satisfied for each TE propagation constant k_3 of a dielectric slab waveguide. It is interesting to see that, contrary to the TM case, $\mathrm{Im}\{\epsilon_{\mathrm{eff}}\}$ is expressed in simple terms of the confinement factors only.

We remark that for the TE_0 mode of symmetric three-layer structures, (19) reduces to an expression which was derived by Buus [5] as a scalar approximation. The same expression can also be found in [6], where it is not mentioned that its validity is restricted to the TE case. It can also be found in [2, ch. 5] where it is derived for both TE and TM under the assumption of weak guiding. Together with an estimation of the confinement factors [7] (also only valid for the TE_0 mode of three-layer waveguides), (19) gives a rule of thumb to estimate $\mathrm{Im}\{\epsilon_{\mathrm{eff}}\}$ for that particular case.

The identities (14) and (19) provide two easily performed checks on the accuracy of mode solvers. As an example, we have checked the results of a recently developed mode solver [1] for the configuration of Fig. 1. It was found that the effective index and the field distributions indeed satisfy (14) and (19) to seven significant digits for both TE_0 and TM_0. Agreement to five significant digits was obtained for higher order modes in more complex structures. This indicates the degree of precision of the mode solver. We noted that the second summation term in (14) can be of the same order of magnitude as the first.

Next, we derive two approximations for the modal gain. Starting from (19), while using $g_{\mathrm{mod}} = 2k_0\mathrm{Im}\{n_{\mathrm{eff}}\}$ and $\epsilon_{\mathrm{eff}} = n_{\mathrm{eff}}^2$, and assuming weak contrast, i.e.,

$$\mathrm{Re}\{n_{\mathrm{eff}}\} \approx \mathrm{Re}\{n_i\} \tag{21}$$

immediately leads to

$$g_{\mathrm{mod}} \approx \sum_i \Gamma_i^{\mathrm{TE}} g_i, \tag{22}$$

with $g_i = 2k_0\,\mathrm{Im}\{n_i\}$. This is a well known result. It can also be derived from a scalar theory starting from the Helmholtz equation for a scalar field. However, contrary to conventional wisdom, such a relation does not hold for TM modes. This is immediately clear from (14). Making the additional assumption that

$$\mathrm{Re}\{n_i\} \gg \mathrm{Im}\{n_i\} \tag{23}$$

it follows that

$$g_{\mathrm{mod}} \approx \sum_i \Gamma_i^{\mathrm{TM}} g_i - (k_0 I)^{-1} \sum_{x_{1,i}} \mathrm{Im}\left\{ \tilde{H}_2^* \frac{j\Delta \mathrm{Im}\{n_i\}}{\mathrm{Re}\{n_i\}} \overline{\partial_1 H_2} \right\}, \tag{24}$$

where $\Delta\mathrm{Im}\{n_i\} = \mathrm{Im}\{n_{i+1}\} - \mathrm{Im}\{n_i\}$. Unlike (22), this is a new result. Just like before, the connection between gain and confinement for TM solutions differs from that for TE modes. Also, the sum in (24) can be of the same order of magnitude as the first term on the right-hand side for realistic configurations. Approximation (24) implies that if one starts out from a scalar theory and uses (22) for the modal gain, then one is really restricting oneself to TE modes alone. In other words, a scalar theory does not amount to 'neglecting polarization effects,' but is an approximation for TE modes, rather than TM modes. Judging from the literature, this point is not generally appreciated.

II. CONCLUSION

We have found two identities for the complex propagation constant for any guided mode of any dielectric slab waveguide. The relations, which have different forms for TM and TE, are in terms of field confinement factors. The confinement factors pertain to $|\tilde{H}_2|^2$ and $|\tilde{E}_2|^2$, rather than the Poynting vector. They can be used to check the accuracy of mode solvers. It was found for several examples that the effective index and the field distribution indeed satisfy the identities.

From these two identies, two approximations for the modal gain were derived. It was shown that an often used approximation which expresses the modal gain in terms of confinement factors, is not valid for TM modes.

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