

Theoretical and experimental analysis of unidirectionality of asymmetrically coupled semiconductor ring or disk lasers

Geert Morthier^{a,c}, Pauline Mechet^{a,c}

^aPhotonics Research Group, INTEC-Department, Ghent University-IMEC, Ghent B-9000, Belgium

^cCenter for Nano- and Biophotonics (NB-Photonics), Ghent University, Ghent B-9000, Belgium

*morthier@intec.ugent.be; phone +32 - 9 - 264 8934; fax +32-9-264 3593;

ABSTRACT

We present analytical, numerical and experimental results about the unidirectional behaviour of semiconductor ring or disk lasers in which the coupling from the clockwise (CW) mode to the counterclockwise (CCW) mode is different from the coupling from the counterclockwise to the clockwise mode. The theoretical and numerical results show different regimes, depending on the gain suppression in the active layer. At very low power, the ratio of the powers of the CW and CCW modes depends mainly on the ratio of the coupling constants, while at high power gain suppression is dominant and the ratio of the powers depends on the ratio of the gain suppression and the weakest coupling coefficient. Our analytical expressions for the unidirectionality are in excellent agreement with numerical results obtained using coupled rate equations. Some experimental results are given for a microdisk laser coupled to a bus waveguide, on one side of which an almost 100% reflecting Bragg grating is designed.

Keywords: semiconductor ring lasers, unidirectional behaviour, feedback sensitivity

1. INTRODUCTION

Semiconductor ring or disk lasers are very well suited for use in photonic integrated circuits, since no reflecting (i.e. cleaved) facets or diffractive gratings are required for their operation. The facts that no facets or gratings are required makes it also relatively easy to fabricate such lasers and, e.g., no overgrowth is required. Moreover, as the cavity losses in these lasers can be made very small, very compact disk and ring lasers have been demonstrated in the recent years [1]. Using the high optical confinement of InP membranes bonded heterogeneously onto silicon-on-insulator, low threshold and very compact microdisk lasers with a diameter of 5 μ m have been demonstrated [2].

Such ring and disk lasers can be used for applications in all-optical signal processing as well and several functionalities have in the meantime been demonstrated with good performance. All-optical set-reset flip-flops with a total power consumption of only a few mW, switching energies of 1.8fJ and switching times of 60ps have been discussed in [3]. In addition, all-optical wavelength conversion, signal regeneration, gating, and format conversion have been demonstrated with the same microdisk lasers and up to bitrates of 10-20 Gbit/s. Similar results have been obtained with all-InP microring lasers [4].

In addition, microdisk lasers are also very well suited for all-optical interconnect implementations. Arrays of microdisk lasers, all with slightly different diameter and coupled to the same bus waveguide, have been demonstrated as multiwavelength light sources that are ideally suited for optical interconnect. Using InP heaters around the microdisk, the wavelengths of the individual disks can be thermally tuned within a limited range to target predefined wavelengths [5]. Moreover, it has also been reported that microdisks can be easily integrated with photodetectors or even used themselves for detection [6,7]. They can also either be directly modulated or used as external modulator.

Depending on their specific properties, ring or disk lasers can operate in either a bidirectional or a unidirectional mode and in some cases the unidirectional operation can be bistable, whereby the laser can operate stably in the clockwise mode or the counterclockwise mode and switching between both modes is possible using optical pulses. For operation with high efficiency in e.g. optical interconnect, operation in a unidirectional mode with predefined direction is

preferred. For the application as optical regenerator, a certain degree of unidirectional operation is also necessary and the decision threshold is determined by this degree of unidirectionality [8].

Unidirectionality is generally possible thanks to a different self and cross gain suppression in ring lasers, but is counteracted by a.o. mutual coupling between clockwise and counter clockwise propagating waves due to scattering, which itself is due to sidewall surface roughness. Asymmetric coupling between clockwise and counter clockwise modes, e.g. resulting from a low reflection at one end of the bus waveguide and a high reflection at the other end, on the other hand will also promote unidirectionality.

There have been several reports recently on unidirectional ring lasers or microdisk lasers coupled to a relatively strong reflector at one side only of the bus waveguide [10,11], such as shown in Figure 1. In [10], such unidirectional operation has been qualitatively demonstrated using camera images of the light scattered by the waveguide roughness. In [11], a more detailed experimental and numerical analysis is presented for heterogeneously integrated microdisk lasers coupled to a bus waveguide on one side of which an almost 100% reflecting Bragg grating was designed.

Not surprisingly, unidirectional operation has also been observed in ring lasers in which a nonreciprocal, magnetically controlled semiconductor optical amplifier was incorporated [13], and in a ring laser coupled with an S-section [12]. Such solutions are however far more complicated than simple ring or disk lasers with an asymmetrically reflecting bus waveguide.

In this paper, we focus on ring or disk lasers coupled to a bus waveguide with at one side a non-reflecting end and at the other side a strongly reflecting end, e.g. due to the presence of a Bragg grating in the bus waveguide. We derive the ratios of clockwise and counter clockwise power in the ring/disk analytically and thus obtain an analytical description of the degree of unidirectionality. We will see that there are two different regimes, a low-power regime where gain suppression only has a minor influence and a high-power regime where gain suppression has a dominant influence.

From the theoretical analysis, it is also possible to determine the external feedback sensitivity of unidirectional ring or disk lasers and to compare the feedback sensitivity (normalized to facet loss) to other laser structures. At low power levels, highly unidirectional ring or disk lasers are more sensitive to external reflections than edge-emitting lasers such as Fabry-Perot or DFB lasers. At high enough power levels though, the gain suppression makes these lasers potentially less sensitive to external feedback. This will be discussed during the oral presentation.

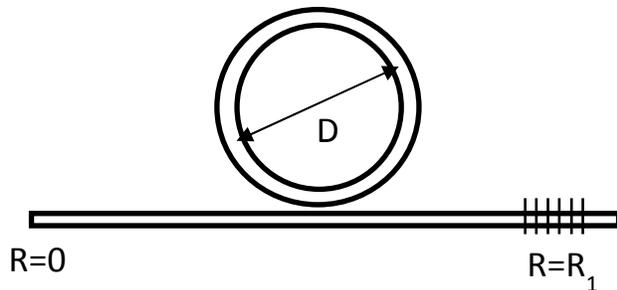


Figure 1. Schematic structure of a ring laser coupled to a bus waveguide, with a reflecting Bragg grating on one side.

2. COUPLED RATE EQUATIONS

We start from the coupled rate equations for the complex field amplitudes E_{CW} and E_{CCW} of the clockwise and counter clockwise propagating laser modes respectively:

$$\frac{dE_{CW}}{dt} = \frac{1}{2}(1 + j\alpha) \left[G - \frac{1}{\tau_p} \right] E_{CW} + K_1 E_{CCW} \quad (1)$$

$$\frac{dE_{CCW}}{dt} = \frac{1}{2}(1 + j\alpha) \left[G - \frac{1}{\tau_p} \right] E_{CCW} + K_2 E_{CW} \quad (2)$$

With

$$E_{CW} = \sqrt{S_{CW}} \exp(j\varphi_{CW}), E_{CCW} = \sqrt{S_{CCW}} \exp(j\varphi_{CCW}), K_i = |K_i| \exp(j\phi_i) \quad (3)$$

The coupling coefficients K_i are the total field reflection (including phase) seen by the CW or CCW mode divided by the roundtrip time of the ring or disk under consideration. They include scattering due to sidewall roughness as well as reflections from facets or gratings in the bus waveguide. We have normalized the optical fields such that their squared amplitude is equal to the photon number S . We can now decompose the equations for the complex electrical fields into an amplitude and phase equation. We consider the static case, for which the field amplitudes (and photon numbers S) are constant in time. Important in ring or disk lasers is that the gain for the CW and CCW mode experiences different gain suppression, i.e.

$$\begin{aligned} G_{CW} &= G_0(N)/[1 + \epsilon S_{CW} + 2\epsilon S_{CCW}] \\ G_{CCW} &= G_0(N)/[1 + 2\epsilon S_{CW} + \epsilon S_{CCW}] \end{aligned} \quad (4)$$

This gain suppression is spectrally symmetric around the laser line, such that there is no effect on the refractive index (or on the phase). Eqs. (1) to (4) can be combined to obtain (with $\Delta\varphi = \varphi_{CW} - \varphi_{CCW}$ and $S_{CW}/S_{CCW} = \lambda^2$):

$$G_0/[1 + \epsilon S_{CW} + 2\epsilon S_{CCW}] = \frac{1}{\tau_p} - 2 \frac{|K_1|}{\lambda} \cos(\Delta\varphi - \phi_1) \quad (5)$$

$$G_0[1 + 2\epsilon S_{CW} + \epsilon S_{CCW}] = \frac{1}{\tau_p} - 2|K_2|\lambda \cos(\Delta\varphi + \phi_2) \quad (6)$$

$$\frac{d\varphi_{CW}}{dt} = \Delta\omega = \frac{\alpha}{2} \left(G_0 - \frac{1}{\tau_p} \right) - \frac{|K_1|}{\lambda} \sin(\Delta\varphi - \phi_1) \quad (7)$$

$$\frac{d\varphi_{CCW}}{dt} = \Delta\omega = \frac{\alpha}{2} \left(G_0 - \frac{1}{\tau_p} \right) + |K_2|\lambda \sin(\Delta\varphi + \phi_2) \quad (8)$$

3. LOW POWER REGIME

In a ring/disk laser coupled to a reflector on one side of the bus waveguide, one can assume that K_2 is due to sidewall roughness and residual facet reflection, while K_1 also includes reflection from the reflector (e.g. Bragg reflector) in the bus waveguide and is thus much larger than K_2 . With κ being the coupling between the ring/disk and the bus waveguide and r_1 the field reflection in the bus waveguide, one can write:

$$K_1 = K_2 + |\kappa|^2 \frac{r_1}{\pi D} v_g \quad (9)$$

D is the diameter of the ring/disk and v_g the group velocity in the laser cavity. r_1 should also include the phase delay due to the coupler and due to the propagation between the ring and the reflector (being a facet or Bragg grating e.g.).

We will first derive λ from the equations (5)-(8) for bias currents close to the threshold current, when the gain suppression can still be neglected. From (7) and (8), one finds that:

$$|K_1| \sin(\Delta\varphi - \phi_1) = -|K_2|\lambda^2 \sin(\Delta\varphi + \phi_2) \quad (10)$$

While subtracting (5) and (6), for $\epsilon=0$, gives:

$$|K_1| \cos(\Delta\varphi - \phi_1) = |K_2|\lambda^2 \cos(\Delta\varphi + \phi_2) \quad (11)$$

Dividing (10) and (11) then results in:

$$\tan(\Delta\varphi - \phi_1) = -\tan(\Delta\varphi + \phi_2), \text{ or } \Delta\varphi = \frac{\phi_1 - \phi_2}{2} + m\pi \quad (m = 0,1) \quad (12)$$

Substituting $\Delta\varphi$ in (10) or (11) readily gives:

$$\lambda^2 = \frac{S_{CW}}{S_{CCW}} = \frac{|K_1|}{|K_2|} \quad (13)$$

Which shows that the powers in the clockwise and counter clockwise modes are in the same ratio as their respective field reflection coefficients. For the threshold gain, one easily finds:

$$G_0 = \frac{1}{\tau_p} - 2\sqrt{|K_1 K_2|} \cos\left(m\pi - \frac{\phi_1 + \phi_2}{2}\right) \quad (14)$$

The value for m (0 or 1) that gives a positive cosine has to be chosen as it is the solution that results in lowest threshold gain. A typical value for K_2 is $6.28 \cdot 10^9 \text{ s}^{-1}$ (see [3]). For a microdisk laser with a $10 \mu\text{m}$ diameter, a group index of 3, 2% of coupling between the disk and the bus waveguide and a reflection $r_1=1$, we find for K_1 the value 0.02

$10^{14} \mu\text{m/s}/(\pi 10 \mu\text{m}) = 6.36 \cdot 10^{10} \text{ s}^{-1}$. This gives a ratio $|K_1/K_2|$ of about 10.

At higher bias currents, one also has to include the gain suppression. Equation (10) doesn't change in this case, but equation (11) must be replaced by (with $\varepsilon' = \varepsilon/(1+3\varepsilon S_{CW})$):

$$\varepsilon' G_0 (S_{CW} - S_{CCW}) = 2|K_2| \lambda \cos(\Delta\varphi + \phi_2) - \frac{2}{\lambda} |K_1| \cos(\Delta\varphi - \phi_1) \quad (15)$$

$$\varepsilon' G_0 (S_{CW} - S_{CCW}) = 2|K_2| \lambda \frac{\sin[2\Delta\varphi + \phi_2 - \phi_1]}{\sin[\Delta\varphi - \phi_1]} \approx \varepsilon' G_0 S_{CW} \quad (16)$$

We now consider the case with $\lambda \gg 1$, i.e. $|K_1| \gg |K_2|$, and thus can neglect S_{CCW} . To take into account the gain suppression, we introduce the approximations:

$$\lambda^2 = \frac{|K_1|}{|K_2|} (1 + \delta) \quad \text{and} \quad \Delta\varphi = \frac{\phi_1 - \phi_2}{2} + \delta\varphi \quad (17)$$

with δ and $\delta\varphi$ being small. Substituting these expansions in (10) allows to derive $\delta\varphi$ and substitution in (17) gives:

$$\varepsilon' G_0 S_{CW} \cos\left(m\pi - \frac{\phi_1 + \phi_2}{2}\right) = 2\delta \sqrt{|K_1| |K_2|} \quad (18)$$

$$\lambda^2 = \frac{|K_1|}{|K_2|} \left(1 + \frac{\varepsilon' G_0 S_{CW} \cos\left(m\pi - \frac{\phi_1 + \phi_2}{2}\right)}{2\sqrt{|K_1| |K_2|}} \right)$$

For a numerical example, we consider again the microdisk with a $10 \mu\text{m}$ diameter, for which $|K_1|$ and $|K_2|$ are $6.36 \cdot 10^{10} \text{ s}^{-1}$ and $6.36 \cdot 10^9 \text{ s}^{-1}$ respectively. In Figure 2, we plot the values of λ^2 obtained for two different values of ε , $\varepsilon = 1 \cdot 10^{-18} \text{ cm}^3$ and $\varepsilon = 2 \cdot 10^{-18} \text{ cm}^3$, and for $(\phi_1, \phi_2) = (0, 0)$ and $(\phi_1, \phi_2) = (0, \pi/2)$ respectively. The solid lines with symbols represent the values obtained from a numerical time domain simulation of the coupled equations (1), while the dashed lines represent the values obtained using (18). The approximation can be made even better by replacing S_{CW} by $S_{CW}(1 - |K_2/K_1|)$.

For the special case $\phi_1 + \phi_2 = \pi$, and for $\varepsilon = 0$, one finds from (11) that $\Delta\varphi - \phi_1 = \pm\pi/2$ and thus $\cos(\Delta\varphi - \phi_1) = 0$. Although one finds from (10) that $\lambda^2 = |K_1/K_2|$, we obtain from the time domain numerical analysis a self-pulsating behaviour irrespective of the value of ε .

It is emphasized that the above approximations are only valid as long as δ is small, i.e. as long as $\varepsilon G_0 S_{CW} < (2\sqrt{|K_1| |K_2|})$.

4. HIGH POWER REGIME

For high power levels or low coupling constants K_1 and K_2 , one has $\varepsilon G_0 S_{CW} \gg (2\sqrt{|K_1| |K_2|})$, and in this case we can assume that $\lambda^2 \gg |K_1|/|K_2|$. From (10), it then follows that:

$$\Delta\varphi + \phi_2 = 0 \quad \text{and} \quad \Delta\varphi - \phi_1 = -(\phi_1 + \phi_2) \quad (19)$$

Substitution in (16) gives:

$$\epsilon'G_0S_{CW} = 2|K_2|\lambda - \frac{2}{\lambda}|K_1|\cos(\phi_1 + \phi_2) \quad (20)$$

with solution:

$$\lambda = \frac{\epsilon'G_0S_{CW} + \sqrt{(\epsilon'G_0S_{CW})^2 - 8|K_2|\epsilon'G_0S_{CW} + 16|K_2K_1|\cos(\phi_1 + \phi_2)}}{4|K_2|} \approx \frac{\epsilon'G_0S_{CW}}{2|K_2|} \quad (21)$$

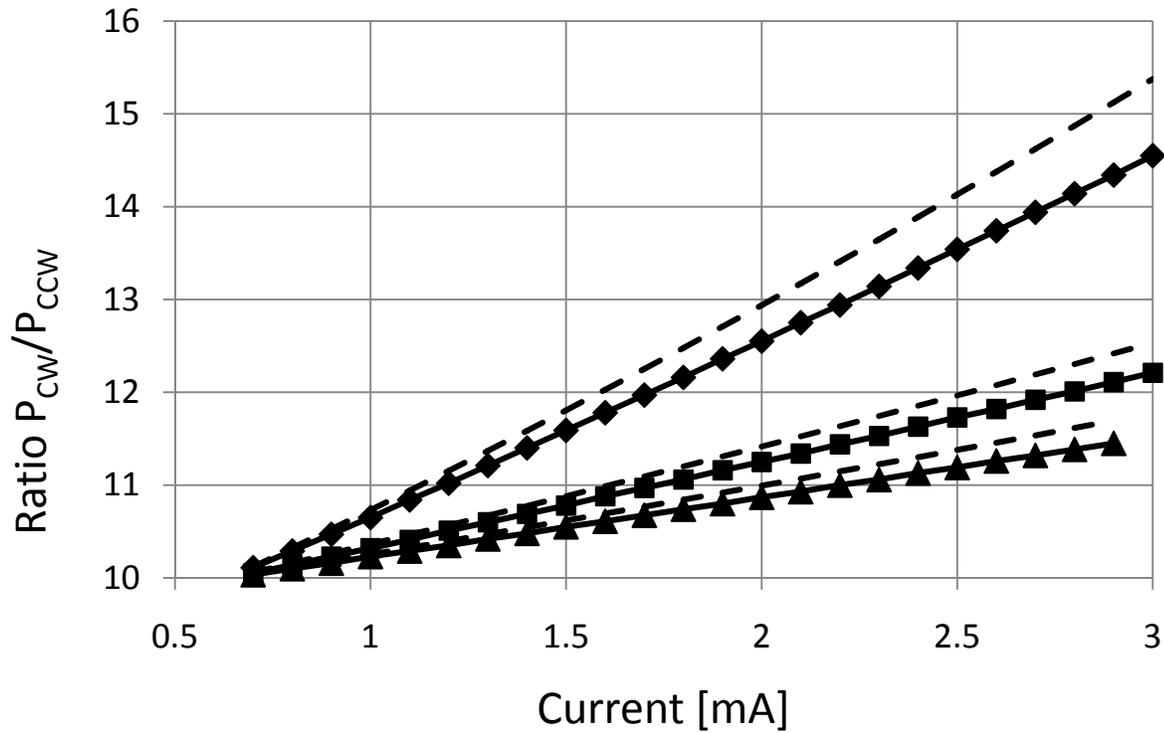


Figure 2. Ratio of power in CW and CCW modes, obtained from a numerical solution of the coupled rate equations, for a microdisk laser with coupling coefficients $K_1=6 \cdot 10^{10} \text{ s}^{-1}$, and $K_2=6 \cdot 10^9 \text{ s}^{-1}$, (◆) for $(\phi_1, \phi_2)=(0, 0)$ and $\epsilon=2 \cdot 10^{-18} \text{ cm}^3$, (■) for $(\phi_1, \phi_2)=(0, 0)$ and $\epsilon=1 \cdot 10^{-18} \text{ cm}^3$, and (▲) for $(\phi_1, \phi_2)=(0, \pi/2)$ and $\epsilon=1 \cdot 10^{-18} \text{ cm}^3$. The dashed lines are the approximation as obtained by (20).

Figure 3 shows λ^2 obtained from a time domain numerical solution of the coupled wave equations (1) up to higher current levels for the case, $K_1=6.36 \cdot 10^8 \text{ s}^{-1}$, $K_2=3.18 \cdot 10^8 \text{ s}^{-1}$, and $\epsilon=1 \cdot 10^{-18} \text{ cm}^3$ as well as the results for currents above 2.5mA obtained using (21). Although the ratio K_1/K_2 is only 2, one obtains much higher λ^2 values and they correspond quite well to the values obtained using (21).

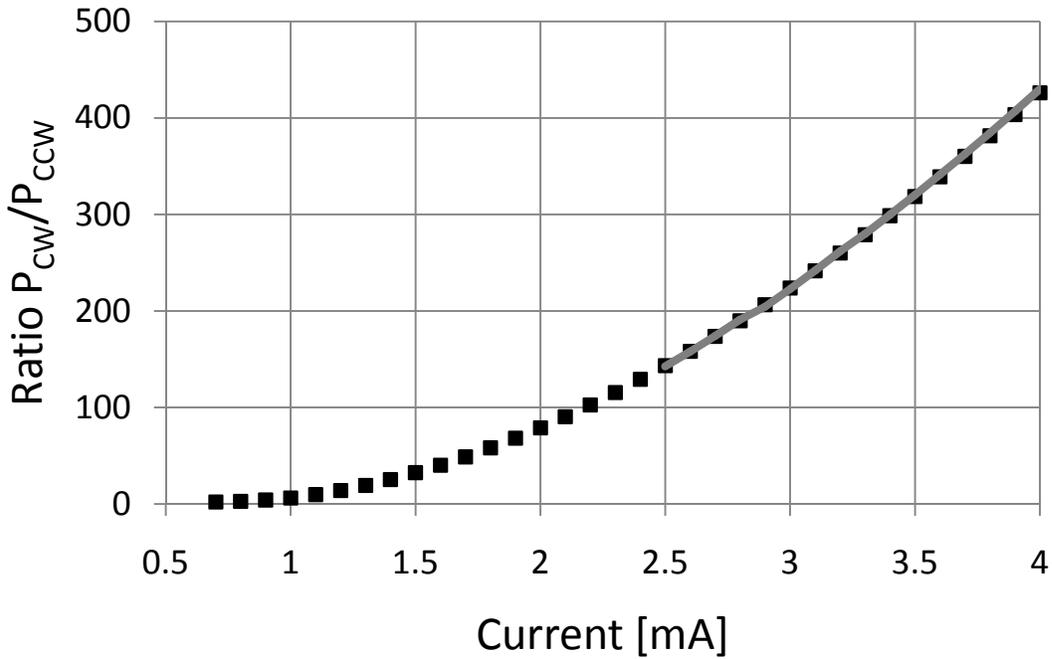


Figure 3. Ratio of power in CW and CCW modes, (■) obtained from a numerical solution of the coupled rate equations, for a microdisk laser with coupling coefficients $K_1=6.36 \cdot 10^8 s^{-1}$, and $3.18 \cdot 10^8 s^{-1}$, and (—) obtained from expression (23) $\epsilon=1 \cdot 10^{-18} cm^3$.

5. EXPERIMENTAL RESULTS

Two microdisk lasers with diameter $7.5\mu m$ and a highly reflecting Bragg mirror on one side were investigated experimentally. Figure 4 shows the measured output power spectra from both output sides for one of these lasers.

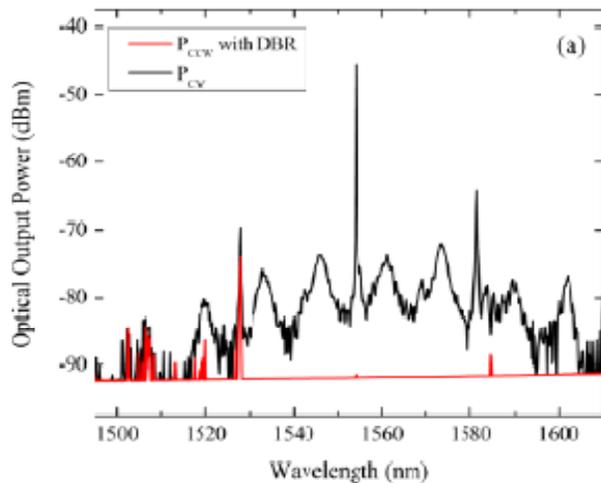


Figure 4. Output spectra from clockwise and counter clockwise mode for a $7.5\mu m$ diameter microdisk laser with Bragg reflector on one side of the bus waveguide.

One can see that the extinction ratio between the optical powers coupled out of the waveguide on the side of the DBR structure and on the side without DBR is 46.1 dB. Measurements on separate DBR mirrors indicated a reduction in transmission of 33.1dB, giving an extinction ratio inside the disk between clockwise and counter clockwise of 13dB. Hence, the microdisk laser behaves as a unidirectional laser with predefined lasing direction and it can be used as a high efficiency light source.

6. CONCLUSION

We have shown theoretically that ring or disk lasers with a strong reflection from one side and weak reflection from the other side can be operating in a unidirectional mode. At low power levels, the ratio of powers in clockwise and counter clockwise mode is approximately equal to the ratio of the coupling coefficients between clockwise and counter clockwise mode, while at high power levels this ratio is determined by gain suppression and the lowest coupling coefficient. A unidirectional operation with predefined direction thus requires only a modest asymmetry if sufficiently high power levels can be reached inside the ring or disk. At lower power levels on the contrary, a strong asymmetry is required.

From the equations for $\Delta\omega$ it is possible to derive sensitivity to external feedback. This will be discussed during the conference.

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