

# Feedback Sensitivity of DBR-Type Laser Diodes

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**Abstract**—It is shown theoretically that the feedback sensitivity of Distributed Bragg Reflector laser diodes with low-loss Bragg section decreases with the length of the Bragg section and also can be decreased by detuning from the Bragg peak. The effect of detuning is not only due to a change in effective linewidth enhancement factor, but also due to a change of the effective cavity length. The results can be applied also to laser diodes coupled on one side to one or more ring resonators. Such lasers can have very weak feedback sensitivity.

**Index Terms**—DBR laser diode, narrow linewidth lasers, tunable lasers, feedback sensitivity.

## I. INTRODUCTION

MANY currently used laser diodes still require an isolator in their package to avoid detrimental effects from external reflections, especially if narrow linewidth is required as is the case for e.g., lasers for coherent communication, for spectroscopy and LIDAR [1]–[2].

The requirement to insert an optical isolator in the laser package complicates the package assembly and adds to the cost, but it also complicates efforts to integrate laser diodes with other optical components. Therefore, there has recently been quite some attention for laser diodes that are less sensitive to external optical feedback.

There are basically 2 measures to quantify the feedback sensitivity of a laser diode. A first measure is the feedback level at which the coherence collapse occurs, a feedback level which has been shown [3] to depend on the effective linewidth enhancement factor  $\alpha_{\text{eff}}$  as  $(1 + \alpha_{\text{eff}}^2)/\alpha_{\text{eff}}^4$ . However, external feedback can have a detrimental effect on the laser linewidth even at feedback levels much below the level where coherence collapse occurs. A second measure to quantify the feedback sensitivity of laser diode is therefore related to how the linewidth is affected by even low external feedback levels. This feedback sensitivity is determined by a feedback coefficient  $C$  [2, chapter 9].

Several recent publications (e.g., [4] and [5]) have described laser diodes with zero or near-zero (effective) linewidth enhancement factor  $\alpha$  and coherence collapse in these laser diodes was shown to occur at significantly elevated feedback levels. In [4], quantum dot material was used with very small material  $\alpha$ -factor, while in [5] the effective  $\alpha$ -factor was reduced for carefully

designed DFB (Distributed Feedback), DBR (Distributed Bragg Reflector) and DR (Distributed Reflector) lasers. The influence of the feedback on linewidth (as well as on tuning) is however expressed by the feedback coefficient  $C$ , which so far has been mainly discussed for single section laser diodes (as in [6]).

It is also known very well that the feedback sensitivity of a laser is proportional with its mirror loss [6]. Reducing the mirror loss to very low values however has long been avoided, as it would decrease the efficiency, depending on the ratio of mirror loss to internal loss, too much. Recently, it has been reported though that laser diodes heterogeneously integrated on silicon can have very low internal loss if the waveguide and laser structure is such that the mode is mainly confined to the silicon with only a minor overlap with the III-V material [7]. Such lasers can be designed to have a very low mirror loss and thus a very weak sensitivity to external feedback, while still possessing a reasonable efficiency.

In this paper we investigate the feedback sensitivity of DBR lasers or generally lasers with a gain section connected to an external, passive reflective filter, such as provided by ring resonators coupled to a reflector. It is shown that the feedback coefficient  $C$  of such lasers can be drastically reduced by using a filter with long delay time, i.e., strong dependence of the reflection phase on the frequency. Although it is known that the feedback parameter is inversely proportional with the roundtrip time in the laser diode [2], this has so far been mainly focused on the roundtrip time in single section laser diodes. Little attention has been given to the fact that this roundtrip time can be made very large in laser diodes consisting of a gain section coupled to long (or effectively long) passive reflectors. Such lasers can be fabricated using heterogeneous integration of III-V on silicon or silicon nitride [8]. It is furthermore pointed out that in the case of detuning from the Bragg wavelength (or more generally the reflective filter peak), the feedback coefficient is not simply found by just replacing  $\alpha$  by  $\alpha_{\text{eff}}$  in the standard expression for  $C$  (as given e.g., in [2]). Detuning can give a significant increase of the roundtrip time in the laser diode and reduce the feedback coefficient  $C$  in this way. This is especially important for longer Bragg sections.

The paper is organized as follows. In Section 2, we derive analytical expressions for the feedback coefficient  $C$  when dispersion in the amplitude and phase of the external reflector are included. We point out that these expressions give different results than what would be obtained by just replacing  $\alpha$  by  $\alpha_{\text{eff}}$  in the standard expression for  $C$  (as given e.g., in [2]). In Section 3, we present numerical results for DBR lasers with identical gain section and lossless Bragg sections with different lengths (while keeping the normalized coupling coefficient constant).

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Fig. 1. Schematic of the DBR laser facing an external reflection as considered in this work.

The Bragg sections are approximated as lossless as this allows simple, straightforward calculations. This is also a relatively good approximation for Bragg sections implemented in Si or SiN on insulator. We consider the different factors, effective  $\alpha$ -factor and cavity roundtrip time, that determine the feedback coefficient  $C$ . In Section 4 we look at the feedback level at which coherence collapse occurs. We conclude in Section 5.

Although detailed results are given in this paper only for 2-section DBR laser diodes with lossless Bragg section, it is straightforward, albeit a bit more tedious, to extend the calculations to lossy Bragg sections or other dispersive reflectors (in particular reflectors consisting of one or more ring resonators coupled to a broadband reflector).

## II. RATE EQUATION ANALYSIS

The configuration considered in this paper is schematically shown in Fig. 1. We use a frequency dependent reflection  $r_2(\omega) = |r_2(\omega)|\exp(-j\phi_2)$  and start from the roundtrip condition below for the solitary laser diode:

$$E(t) = E(t - \tau_a) \sqrt{R_1 R_2(\omega)} \exp(-j\phi_2) \exp(-2jk n_a L_a) \exp[(\Gamma g - \alpha_{int}) L_a] \quad (1)$$

with  $\tau_a$  the roundtrip time ( $= 2L_a/v_{ga}$ ) in the gain section with length  $L_a$  and  $n_a$ ,  $\Gamma g$ ,  $\alpha_{int}$  the real effective index, the modal gain per cm and the internal loss in the gain section. The passive section, whether Bragg section or a structure consisting of one or more rings, is assumed to give a frequency dependent power reflection  $R_2(\omega)$  with frequency dependent phase  $\phi_2(\omega)$ . Expressing  $E(t)$  as  $\sqrt{S(t)}\exp[j\varphi(t) + j\omega_0 t]$ , with  $S$  the number of photons inside the gain section, gives the following rate equations for  $S(t)$  and  $\Delta\omega = d\varphi/dt$ :

$$\begin{aligned} \frac{dS}{dt} &= [G(N) - \gamma(\omega)] S(t) \\ \Delta\omega &= \frac{\alpha}{2} \frac{dG}{dN} \Delta N - \frac{1}{\tau_a} \frac{\partial\phi_2}{\partial\omega} \Delta\omega \end{aligned} \quad (2)$$

These equations can be extended to include an external reflection  $R_e$ , faced by the front facet with reflectivity  $R_1$ . We add an extra term to the roundtrip condition that represents the external feedback  $\eta E(t - \tau_{ext})$  facing the facet with reflection  $R_1$ :

$$E(t) = E(t - \tau_a) \sqrt{R_1 R_2(\omega)} \exp(-j\phi_2) \exp(-2jk n_a L_a) \exp[(\Gamma g - \alpha_{int}) L_a] + \eta E(t - \tau_{ext}) \quad (3)$$

With:

$$\eta = \frac{1 - R_1}{\sqrt{R_1}} \sqrt{R_e} = \frac{1 - R_1}{\sqrt{R_1}} r_e \quad (4)$$

Without Langevin functions, representing the noise, this results in the following rate equations.

$$\begin{aligned} \frac{dS}{dt} &= [G(N) - \gamma(\omega)] S(t) + \frac{2\eta}{\tau_a} \sqrt{S(t) S(t - \tau_{ext})} \\ &\quad \cos(\varphi(t) - \varphi(\tau_{ext}) + \omega_0 \tau_{ext}) \\ \Delta\omega &= \frac{\alpha}{2} \frac{dG}{dN} \Delta N - \frac{1}{\tau_a} \frac{\partial\phi_2}{\partial\omega} \Delta\omega + \frac{\eta}{\tau_a} \sqrt{\frac{S(t - \tau_{ext})}{S(t)}} \\ &\quad \sin(\varphi(t) - \varphi(\tau_{ext}) + \omega_0 \tau_{ext}) \end{aligned} \quad (5)$$

From (5), one can derive an expression for  $\omega$  in the static case ( $S(t) = S(t - \tau_{ext})$  and  $\varphi(t) - \varphi(\tau_{ext}) = \Delta\omega \tau_{ext}$ ):

$$\omega \tau_{ext} = \omega_0 \tau_{ext} + \frac{\frac{\eta}{\tau_a} \sqrt{1 + \alpha^2} \tau_{ext}}{1 + \frac{1}{\tau_a} \frac{\partial\phi_2}{\partial\omega} - \frac{\alpha}{2} \frac{\partial\gamma}{\partial\omega}} \cdot \sin(\omega \tau_{ext} + \arctg(\alpha)) \quad (6)$$

Which means that the feedback coefficient  $C$ , as defined in [2], is given by:

$$C = \frac{\frac{\eta}{\tau_a} \sqrt{1 + \alpha^2} \tau_{ext}}{1 + \frac{1}{\tau_a} \frac{\partial\phi_2}{\partial\omega} - \frac{\alpha}{2} \frac{\partial\gamma}{\partial\omega}} = \frac{\frac{\eta}{\tau_a} \sqrt{1 + \alpha^2} \tau_{ext}}{1 + \frac{1}{\tau_a} \frac{\partial\phi_2}{\partial\omega} + \frac{1}{\tau_a} \frac{\alpha}{2} \frac{\partial \ln(R_2(\omega))}{\partial\omega}} \quad (7)$$

i.e., it is modified by a factor  $1 + \frac{1}{\tau_a} [\frac{\partial\phi_2}{\partial\omega} + \frac{\alpha}{2} \frac{\partial \ln(R_2(\omega))}{\partial\omega}]$  as compared to the feedback coefficient of an equivalent Fabry-Perot laser diode with the facet reflectivities  $R_1$  and  $R_2(\omega_0)$ . This reduction can result from a Bragg and phase section, but also from one or more ring resonators (and reflectors). The reduction factor  $1 + \frac{1}{\tau_a} [\frac{\partial\phi_2}{\partial\omega} + \frac{\alpha}{2} \frac{\partial \ln(R_2(\omega))}{\partial\omega}]$  is identical to the factor found in [9, eqs. 14-17] for the modification of the linewidth.

By adding Langevin functions to the rate equations (5), one can readily derive the linewidth  $\Delta\nu$  of the laser diode:

$$\Delta\nu = \frac{\Delta\nu_0}{\left\{ 1 + \frac{\frac{\eta}{\tau_a} \sqrt{1 + \alpha^2} \tau_{ext}}{1 + \frac{1}{\tau_a} \frac{\partial\phi_2}{\partial\omega} - \frac{\alpha}{2} \frac{\partial\gamma}{\partial\omega}} \cos(\omega \tau_{ext} + \arctg(\alpha)) \right\}^2} \quad (8)$$

with  $\Delta\nu_0$  the linewidth of the DBR laser without the external feedback, which is itself also affected by the dispersion in phase and amplitude of the Bragg reflection in the same way as  $C$  [9]:

$$\Delta\nu_0 = \frac{R_{sp} (1 + \alpha^2)}{4\pi S \left\{ 1 + \frac{1}{\tau_a} \frac{\partial\phi_2}{\partial\omega} - \frac{\alpha}{2} \frac{\partial\gamma}{\partial\omega} \right\}^2} \quad (9)$$

with  $R_{sp}$  the spontaneous emission rate. Thus the reduction in feedback sensitivity due to the dispersion also results in a lower linewidth of the laser without feedback.

It is noteworthy that the reduction factor can be related to the longitudinal confinement factor  $\Gamma$  as defined in [10]:

$$1 + \frac{1}{\tau_a} \left[ \frac{\partial \phi_2}{\partial \omega} + \frac{\alpha}{2} \frac{\partial \ln(R_2(\omega))}{\partial \omega} \right] = \text{Re} \left( \frac{1}{\Gamma_z} \right) + \alpha \text{Im} \left( \frac{1}{\Gamma_z} \right) \quad (10)$$

With  $\Gamma_z = \Gamma' + j\Gamma''$  and  $\alpha_{eff} = \frac{\Gamma' \alpha + \Gamma''}{\Gamma' - \alpha \Gamma''}$ , one can convert the expression for C into:

$$C = \frac{\eta \tau_{ext}}{\tau_a} \sqrt{1 + \alpha_{eff}^2} \sqrt{(\Gamma')^2 + (\Gamma'')^2} \quad (11)$$

When the emission is at the Bragg wavelength (or generally at the peak wavelength of the reflection),  $\Gamma'' = 0$ ,  $\alpha_{eff} = \alpha$  and one can define  $\Gamma'$  as:

$$\frac{1}{\Gamma'} = 1 + \frac{1}{\tau_a} \frac{\partial \phi_2}{\partial \omega} = \frac{\tau_t}{\tau_a} \quad (12)$$

With  $\tau_t$  the roundtrip time in the compound cavity. In this case:

$$C = \frac{\eta \tau_{ext}}{\tau_t} \sqrt{1 + \alpha^2} = \frac{\eta \tau_{ext}}{\tau_t} \sqrt{1 + \alpha^2} \quad (13)$$

When the laser is detuned from the peak reflection wavelength, one finds:

$$C = \frac{\eta \tau_{ext}}{\tau_t \rho} \sqrt{1 + \alpha_{eff}^2}, \text{ with } \rho = \sqrt{1 + \left( \frac{1}{2\tau_t} \frac{\partial \ln(R_2)}{\partial \omega} \right)^2} \quad (14)$$

### III. NUMERICAL RESULTS FOR A 2-SECTION DBR LASER DIODE

In this part, we have a look at the numerical values of the different factors  $\tau_t/\tau_a$  and  $\alpha_{eff}$  for 2-section DBR laser diodes with lossless, AR-coated Bragg section ( $R_3 = 0$ ). For such a laser diode, the quantities can readily be derived from the expression for the field reflection of a lossless, AR-coated Bragg section with coupling coefficient  $\kappa$  and length  $L_B$ :

$$r = \frac{j\kappa \sinh(\gamma L_B)}{\gamma \cosh(\gamma L_B) + j\Delta\beta \sinh(\gamma L_B)}, \gamma = \sqrt{\kappa^2 - (\Delta\beta)^2} \quad (15)$$

With  $\Delta\beta = 2\pi n_e/\lambda - \pi/\Lambda$ , with  $n_e$  the effective index,  $\lambda$  the wavelength and  $\Lambda$  the grating period. An analytical expression for  $\tau_t/\tau_a$  can readily be derived from (15). With  $v_{gB}$  the group velocity in the Bragg grating:

$$\frac{\tau_t}{\tau_a} = 1 + \frac{L_B v_{ga}}{2L_a v_{gB}} \frac{\left\{ \frac{\tanh(\gamma L_B)}{\gamma L_B} \left( 1 + \frac{(\Delta\beta L_B)^2}{(\gamma L_B)^2} \right) - \frac{(\Delta\beta L_B)^2}{(\gamma L_B)^2} (1 - \tanh^2(\gamma L_B)) \right\}}{1 + \left( \frac{\Delta\beta}{\gamma} \tanh(\gamma L_B) \right)^2} \quad (16)$$

And at the Bragg wavelength, for which  $\Delta\beta = 0$  and  $\gamma = \kappa$ :

$$\frac{\tau_t}{\tau_a} = 1 + \frac{L_B v_{ga}}{2L_a v_{gB}} \frac{\tanh(\kappa L_B)}{\kappa L_B} = 1 + \frac{v_{ga}}{2L_a v_{gB}} \frac{\tanh(\kappa L_B)}{\kappa} \quad (17)$$

Although the total roundtrip time doesn't directly depend on the length of the Bragg grating, there is an indirect dependence. Indeed, to maintain a low threshold gain and current,

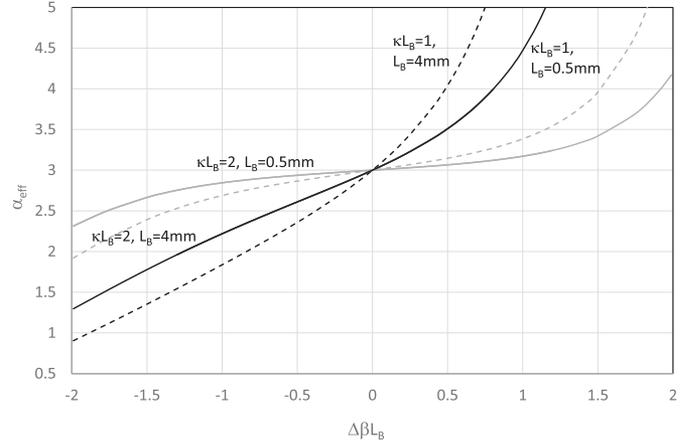


Fig. 2. Effective  $\alpha$ -factor vs normalized detuning  $\Delta\beta L_B$  for DBR laser diodes with  $L_a = 200 \mu\text{m}$ , for 2 different values of  $\kappa L_B$  (1 and 2) and 2 different Bragg section lengths (500  $\mu\text{m}$  and 4000  $\mu\text{m}$ ).

$\kappa L_B$  must be sufficiently large. Hence, to have a large total roundtrip time inside the laser, while also having a low threshold current,  $\kappa$  must be small and  $L_B$  must be correspondingly large. In this case,  $\kappa$  must also be small to guarantee single mode operation.

As an example, we consider two values for the normalized coupling coefficient of the Bragg section:  $\kappa L_B = 1$  and  $\kappa L_B = 2$ , each time with a Bragg section length  $L_B$  of 500 and 4000  $\mu\text{m}$  long. The length of the active layer  $L_a$  was taken as 200  $\mu\text{m}$  and the front facet power reflectivity of the gain section was 20%. We first consider operation at the Bragg wavelength; in this case  $\alpha = \alpha_{eff}$ . The effective length of the Bragg section is  $\tanh(\kappa L_B)/2\kappa$ , with  $L_B$  the physical length of the Bragg section. From (13) it follows easily that C is reduced by a factor  $1 + v_{gB} \tanh(\kappa L_B)/(2\kappa L_a v_{ga})$  with respect to a Fabry-Perot laser with equal facet reflectivities. Assuming the group velocities  $v_{gB}$  and  $v_{ga}$  in gain and Bragg section to be equal, one finds that for  $\kappa L_B = 1$  C is inversely proportional to  $1 + 0.76 L_B/L_a$ , whereas for  $\kappa L_B = 2$  C is inversely proportional to  $1 + 0.48 L_B/L_a$ . As an example, if we consider DBR laser diodes with 500  $\mu\text{m}$  and 4000  $\mu\text{m}$  long Bragg sections (but with  $\kappa L = 1$ ), then the laser with 4000  $\mu\text{m}$  long Bragg section has a feedback coefficient which is 5.6 times smaller than that of the laser with 500  $\mu\text{m}$  long Bragg section. Since C is proportional with the external field reflection  $r_e$ , this implies that the DBR laser with 4000  $\mu\text{m}$  long Bragg section can sustain 15 dB more external feedback for a given C coefficient than the laser with 500  $\mu\text{m}$  long Bragg section.

The influence of detuning from the Bragg peak is found from the changes in effective  $\alpha$ -factor,  $\rho$  and  $\tau_t$  with detuning. The correction factor  $\rho$  was found to be very close to 1 (within less than 1%) for  $\kappa L_B = 2$  and a detuning  $|\Delta\beta L_B| < \kappa L_B$ . For  $\kappa L_B = 1$ ,  $\rho$  varied between 1 and 1.15 for  $L_B = 4000 \mu\text{m}$  and between 1 and 1.06 for  $L_B = 500 \mu\text{m}$ . Both the effective  $\alpha$ -factor and  $\tau_t$  depend significantly on the detuning. Fig. 2 gives the calculated values of effective  $\alpha$ -factor vs. normalized detuning  $\Delta\beta L_B$ , for the cases  $\kappa L_B = 1$  and  $\kappa L_B = 2$ , each time for the 2 different Bragg section lengths of 500 and 4000  $\mu\text{m}$ .  $\alpha_{eff}$  is somewhat

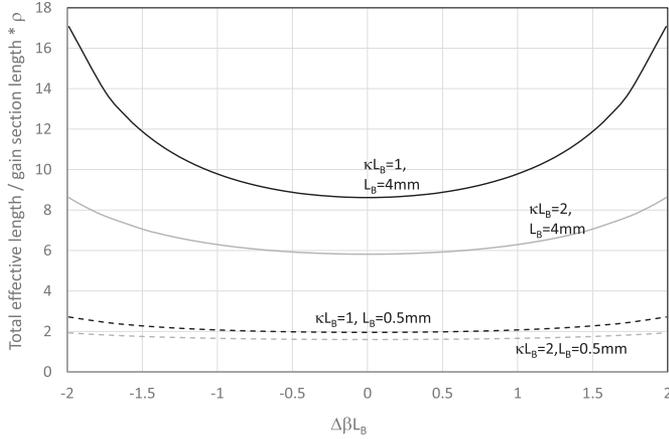


Fig. 3. Total effective cavity length (relative to the length of the gain section) times  $\rho$  vs. the normalized detuning for DBR laser diodes with  $L_a = 200 \mu\text{m}$ , for 2 different values of  $\kappa L_B$  (1 and 2) and 2 different Bragg section lengths (500  $\mu\text{m}$  and 4000  $\mu\text{m}$ ).

dependent on the length of the Bragg section, and its possible reduction is larger for smaller  $\kappa L_B$ .

The detuning also changes the effective length of the Bragg section and  $\tau_t$  significantly. Fig. 3, showing the calculated  $\tau_t/\tau_a \cdot \rho$  vs. normalized detuning, illustrates this. For a 4000  $\mu\text{m}$  long Bragg section with  $\kappa L_B = 1$ , a detuning  $\Delta\beta L_B$  equal to 2 results in about twice the (corrected) effective cavity length than a zero detuning. While the reduction of  $\alpha_{\text{eff}}$  from 3 to 1 gives a reduction of the feedback coefficient with a factor of around 3, the increase of  $\rho\tau_t$  gives another reduction with a factor 2. Together they give a reduction of the feedback coefficient with 6; implying that 15 dB more feedback can be sustained for the same C. For a 4000  $\mu\text{m}$  long Bragg section with  $\kappa L_B = 2$ , the increase of  $\tau_t$  with detuning is about 50%, which is about the same as the decrease of  $\alpha_{\text{eff}}$ .

#### IV. COHERENCE COLLAPSE

Using the numerical simulation tool VPI Photonics Design Suite [11], we investigated also the critical feedback levels at which the onset of coherence collapse starts. For this investigation, we considered just the DBR with  $\kappa L = 1$ , with an external reflection at a distance of 5 cm (corresponding with a delay time of 0.5ns). The onset of coherence collapse was investigated by looking at the Relative Intensity Noise (RIN), averaged between 0.1 and 3 GHz. We considered DBR lasers with Bragg section length of 500 and 4000  $\mu\text{m}$ , emitting close to the Bragg wavelength, as well as the DBR laser with 500  $\mu\text{m}$  long Bragg section but emitting at about 35 GHz negative detuning from the Bragg wavelength. In the numerical simulations, the emission frequency was set by choosing the front facet field reflection phase. In all cases, the gain section was pumped with a bias current of 150 mA. Fig. 4 shows the average RIN values vs. the external feedback level for the 3 DBR laser diodes.

One can see that for the non-detuned DBR laser with 4 mm long Bragg section the RIN starts to increase at a feedback level

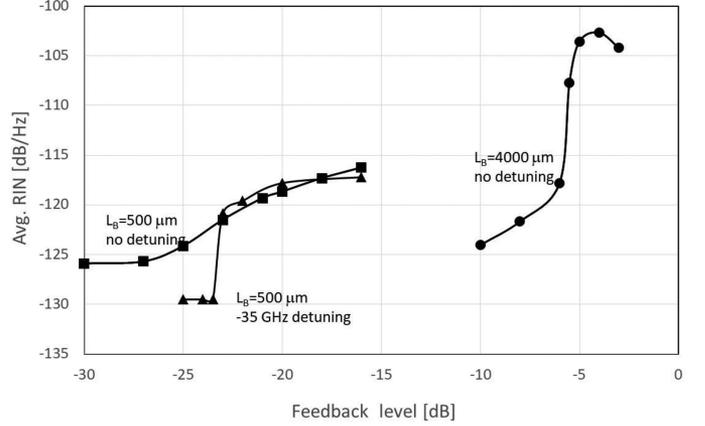


Fig. 4. Total Average RIN between 0.1 and 3 GHz vs. external feedback level for DBR lasers with 200  $\mu\text{m}$  gain section and lossless Bragg section with  $\kappa L = 1$  and Bragg section length of 500  $\mu\text{m}$  and 4 mm. The roundtrip time in the external cavity is 0.5 ns. ( $\alpha = 3$ ).

that is about 15 dB higher than for the non-detuned DBR laser with 500  $\mu\text{m}$  long Bragg section, although the increase with feedback level is slower for the DBR laser with 500  $\mu\text{m}$  Bragg section. It was found furthermore that the shorter laser exhibits an optical spectrum consisting of many lines already at  $-30$  dB feedback, while the longer laser still had a good single mode spectrum at  $-8$  dB feedback. This indicates that the laser with longer Bragg section goes into coherence collapse at a feedback level that is more than 20 dB higher than for the laser with shorter Bragg section. The 15 dB difference in feedback level where the RIN starts to increase corresponds well with the expression for the critical feedback level given in [3]:

$$R_{\text{ext},c} \sim \tau_t^2 \frac{1 + \alpha_{\text{eff}}^2}{\alpha_{\text{eff}}^4} \gamma^2 \quad (18)$$

with  $\gamma\tau$  the damping factor of the relaxation oscillations. Since  $\alpha = \alpha_{\text{eff}}$  at the Bragg wavelength and assuming  $\gamma$  to be quite independent of the Bragg section length, one finds from (16) a critical feedback level proportional with  $(\tau_t)^2$  and thus a critical feedback level that is  $(5.6)^2$  times or 15 dB higher for the laser with 4000  $\mu\text{m}$  long Bragg section than for the laser with 500  $\mu\text{m}$  long Bragg section.

The DBR laser with 500  $\mu\text{m}$  long Bragg section, which was tuned to emit at  $-35$  GHz detuning from the Bragg wavelength, reaches coherence collapse at a feedback level which is around 6 dB higher than for the non-detuned DBR laser with 500  $\mu\text{m}$  long Bragg section. Moreover, this laser had still a good single mode spectrum at a feedback level of  $-24$  dB. A detuning of 35 GHz correspond with a  $\Delta\beta L_B$  of approx.  $-\pi/3$ , giving for  $L_B = 500 \mu\text{m}$  an effective length which is 50% higher than at the Bragg wavelength. Together with the reduction in  $\alpha_{\text{eff}}$  this gives according to (17) around 6 dB higher critical feedback level for the detuned DBR laser.

It is remarked that the analytical formulae which have been reported in literature for the critical feedback level have not been derived from the expression for the feedback coefficient [3], but rather from fitting of numerical results for the RIN.

## V. APPLICATION TO RELATED LASER DIODES

In Sections 3 and 4, the feedback sensitivity has been discussed for 2-section DBR laser diodes. However, it should be clear that the addition of a passive phase section with length  $L_p$  will result in an extra factor  $\exp(-2jkn_p L_p)$  to the reflection in (15), and thus a term  $\frac{2L_p}{v_{gp}}$  to  $\frac{\partial\phi_2}{\partial\omega}$ . And this will reduce the feedback sensitivity even more to a lesser or bigger extent depending on  $L_p$ .

As written before, the results can also be generalized to lasers consisting of a gain section coupled to a reflector including one or more ring resonators, as is the case in many tunable or low linewidth lasers ([10]–[11]). For a single ring resonator at resonance  $\omega_0$ , the effective single pass phase is  $(\frac{1}{2} + \frac{1-\kappa^2}{\kappa^2})\frac{2\pi R(\omega-\omega_0)}{v_g}$ , and thus  $\frac{\partial\phi_2}{\partial\omega} = (\frac{1}{2} + \frac{1-\kappa^2}{\kappa^2})\frac{2\pi R}{v_g}$ , with  $R$  the radius of the ring and  $\kappa$  the coupling between ring and bus waveguides. For small coupling  $\kappa$ , the factor  $(\frac{1}{2} + \frac{1-\kappa^2}{\kappa^2})$  can be very large, and even if  $R$  is not that large, it can cause  $\frac{\partial\phi_2}{\partial\omega}$  to be very large. For structures in which the light passes back and forth through 2 rings, the value can be multiplied with 4. For ring resonators with radius 25  $\mu\text{m}$ , e.g., and coupling of, e.g., 4%, the effective length is 3.9 mm. If there is appropriate detuning between laser wavelength and ring resonances, one can in addition also take advantage of a further increase of the delay time and a modification of the  $\alpha_{\text{eff}}$ .

## VI. CONCLUSION

We have shown that lasers consisting of a gain section coupled to a passive wavelength or frequency dependent reflector can have a very weak feedback sensitivity if the reflector's amplitude and phase have the right dispersion. Many tunable and/or narrow linewidth laser diodes (but also, e.g., semiconductor Fano lasers [13]) are of this type and can thus be designed to be much more resistant to external feedback. We have also shown that such lasers with long (or effectively long) passive reflectors experience coherence collapse at much higher feedback levels. The influence of the passive section length and detuning is not simply explained by the variations in effective linewidth enhancement factor, but also by the large variations in effective cavity length.

The DBR-type lasers can in principle also be fabricated using quantum dot material with low  $\alpha$ -factor. And, especially if integrated on silicon-on-insulator, they can also be designed to have low internal loss and low mirror loss to decrease the feedback sensitivity more. For lasers heterogeneously integrated

on silicon or silicon nitride on insulator, one can implement the reflector using very high Q ring resonators, with a very large effective delay, giving a very large reduction of the feedback sensitivity. In the case of silicon-on-insulator, care must be taken to avoid nonlinear effects though.

Finally, it is emphasized that the approximations introduced in this paper assume very low losses in the passive sections, as is e.g., the case in Si or SiN on insulator waveguides.

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